DOCUMENT RESUME

ED 323 742 EC 232 174

AUTHOR Scheid, Karen
TITLE Cognitive-Based Methods for Teaching Mathematics to Special Education Students: Implications for Media and Material Design. The Instructional Methods Report Series.
INSTITUTION LINC Resources, Inc., Columbus, OH. Information Center for Special Education Media and Materials.
SPONS AGENCY Special Education Programs (ED/OSERS), Washington, DC.
PUB DATE 90
CONTRACT 300-86-0114
NOTE 55p.; For a related document, see EC 232 175.
AVAILABLE FROM LINC Resources, 4820 Indianola Ave., Columbus, OH 43214 ($5.00).
PUB TYPE Guides - Non-Classroom Use (055)

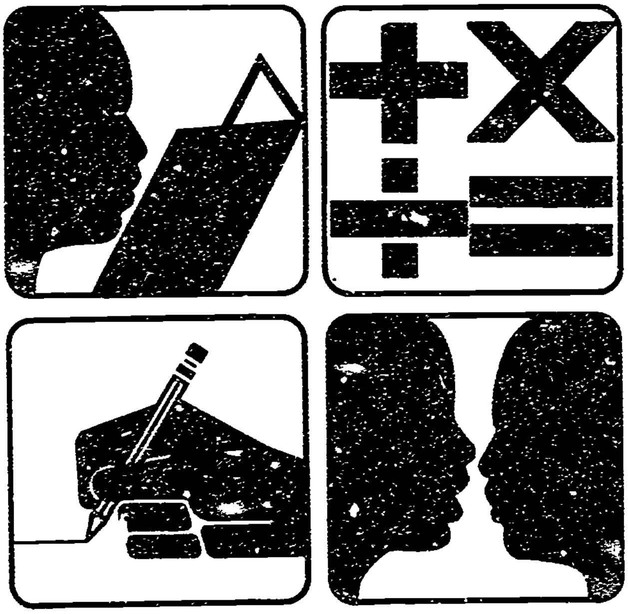
EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS *Cognitive Development; *Concept Formation; Educational Media; Elementary Secondary Education; *Instructional Materials; *Learning Disabilities; Learning Problems; Material Development; *Mathematics Instruction; Mental Retardation; Mild Disabilities; Number Concepts; *Teaching Methods

ABSTRACT

Based on suggestions of participants at the second annual Instructional Methods Forum held in June 1989, the monograph considers cognitive-based approaches to teaching mathematics to students with learning problems and the implications for the design of media and materials. It discusses characteristics of successful cognitive approaches and the role of media and materials in facilitating this form of instruction. An introductory chapter looks at possible reasons for United States students' difficulties with mathematics and proposes that cognitive-based approaches be used to stress conceptual as well as procedural knowledge. The second chapter looks at mathematical learning among students with disabilities. Cognitive-based principles for teaching mathematics are presented in chapter 3, including findings from research on children's mathematical thinking and a taxonomy of word problem types. The fourth chapter considers instructional components of cognitive-based mathematics teaching, grouped into those relating to the content for instruction and those relating to the methods for teaching the content. In the concluding chapter, publishers are encouraged to develop cognitive-based textbooks and instructional materials. Appended are a list of the forum participants, sample records from the instructional materials database of the Information Center for Special Education Media and Materials, and a bibliography of approximately 170 items. (DB)

THE INSTRUCTIONAL METHODS REPORT SERIES

Cognitive-based Methods for Teaching Mathematics to Special Education Students-- Implications for Media and Material Design



Information Center for SPECIAL EDUCATION MEDIA and MATERIALS

Cognitive-based Methods for Teaching Mathematics to Special Education Students-- Implications for Media and Material Design

Prepared by

Karen Scheid
Research Specialist
Information Center for Special Education Media and Materials

1990

This document was prepared by LINC Resources, Inc., 4820 Indianola Avenue, Columbus, Ohio 43214, pursuant to contract 300-86-0114 with the U.S. Department of Education, Office of Special Education Programs. Contractors undertaking such projects under government sponsorship are encouraged to express their judgment freely in professional and technical matters. Points of view or opinions do not, therefore, necessarily reflect the position or policy of ED/OSEP and no official endorsement of these materials should be inferred.

PLEASE NOTE

Permission is granted to reproduce this document for use in professional, education-related activities. However, individuals planning to do so must complete and submit the "Report of Planned Use" form enclosed. For contractual purposes, the Information Center for Special Education Media and Materials needs to maintain records of the uses made of this report.

ACKNOWLEDGEMENTS

The author wishes to thank all of the participants of the 1989 Instructional Methods Forum sponsored by the Information Center for Special Education Media and Materials for their insights and opinions, which provided much of the content for this report. Special thanks are due to the following Forum participants who reviewed and critiqued sections of this report: Arthur Baroody, University of Illinois; John Cawley, State University of New York at Buffalo, Elizabeth Fennema, University of Wisconsin; Sheridan Osterstrom, Buffalo Board of Education, and Diane Rivera, Florida Atlantic University.

Appreciation is extended to the United States Department of Education, Office of Special Education Programs, Division of Innovation and Development, for its financial sponsorship of the Information Center for Special Education Media and Materials and to Beatrice Birman, Chief of the Research and Development Projects Branch, and Doris Cargile, Educational Program Specialist, for their ongoing guidance, advice, and support.

Finally, Judy Bielanski and Linda Bater are due considerable thanks for their conscientious production and editorial assistance.

PREFACE

The Information Center for Special Education Media and Materials is a project of the United States Department of Education's Office of Special Education Programs. Housed at LINC Resources in Columbus, Ohio, the Center's mission is to increase the quality, availability and use of special education media and materials. Specifically, the Center hopes 1) to increase the quantity of media and materials that are designed according to instructional principles, which have proved to be effective with special education populations and 2) to identify ways in which these and other media and materials can best be used to further learning opportunities for children with disabilities.

We know that 90% or more of a student's classroom time is spent with media and materials, yet such materials are but one component of the instructional process. Learner characteristics, expected outcomes, teacher effectiveness, administrative support, the learning environment, educational philosophy, and instructional methods also contribute to positive or negative educational experiences. Any meaningful effort to improve media and materials must take place within the larger context of improvement of instruction. Therefore, the Center must pursue its goal by identifying instructional methods that are effective with youngsters who have disabilities, investigating the factors that make these methods work in the classroom, and specifying the roles that media and materials can play to facilitate instruction in these methods.

The Center's role, then, is to provide leadership by focusing the attention of practitioners, publishers, and researchers on the major issues and questions related to improving the design and use of media and materials. Annually, the Center convenes members of the research, school, and publishing communities to think together, addressing identified issues and questions. Much of this current report is based on the perceptions and suggestions of the participants of the Center's second annual Instructional Methods Forum held in Washington, D.C. in June, 1989. The purpose of the 1989 Forum was to engage the attendees from the higher education, school, and publishing communities in conversations of general issues surrounding the classroom use of cognitive-based approaches for instructing students with learning problems in mathematics, to identify general characteristics of successful cognitive approaches, and to examine the role of media and materials in facilitating this form of instruction. The Forum was successful in surfacing insightful and sometimes divergent opinions, which are reflected throughout this report. We at the Center believe that only through reliance on the wisdom and perspectives of researchers, practitioners, and publishers can we hope to encourage refinement of promising methods, accelerate the incorporation of proved principles into instructional products, and foster the appropriate and effective use of these methods by classroom teachers.

CONTENTS

CHAPTER ONE

Mathematics Instruction Under Examination

The Mathematics Performance of American Youth--A Cause for Concern?

Few people would disagree that a goal of schooling should be the development of young people's understanding of basic mathematical concepts and procedures. All students, including those with learning problems, need to acquire the knowledge and skills that will enable them to "figure out" math-related problems encountered daily at home and in future work situations. But are American youth gaining needed problem solving proficiencies? Results of national testing programs such as the National Assessment of Educational Progress (NAEP) indicate that while American students do well on whole-number computations, they have difficulties with fractions, decimals, and percents and with problems that pose unfamiliar, nonroutine tasks. Word problems that involve two or more steps are particularly problematic for these students (Kouba et al., 1988a). And, although American youth possess a fairly good knowledge of *procedures* associated with rational numbers, probability, measurement, and data organization and interpretation, they lack the conceptual knowledge that enables them to *apply* their knowledge in problem-solving situations (Brown et al., 1988a; Brown et al., 1988b).

Interestingly, other studies suggest that the shortcomings in mathematics performance evidenced among American young people are not universal. The Educational Testing Service (1989) reports on a recent study comparing the mathematics and science performance of 13-year-olds from Canada, Korea, Spain, the United Kingdom, and the United States. American youth scored *last* in mathematical knowledge. Particularly difficult for U.S. students were items requiring the application of intermediate-level math skills in solving two-step word problems. Only 40% of American youngsters as compared to 78% of Korean youth could solve such problems.

Results from studies such as these have led some to conclude that American education is good at teaching students mathematical skills, but falls short in helping youngsters understand the concepts that underlie those skills (Baroody, 1987). Without such understanding, it is unlikely that young people can make appropriate use of the skills and procedural knowledge that they do possess (Baroody, 1989a; Baroody, in press).

Reasons for U.S. Students' Problems with Math--Some Speculations

Many educators, researchers, and curriculum developers have speculated as to the reasons for the poor showing of American youth on mathematics assessments. Some contend that current curricular emphases and teaching methods that stress computation and "getting the right answer quickly" contribute to the depressed state of mathematical functioning among U.S. youngsters. It is argued that traditional instruction pays little attention to developing students' abilities to think mathematically, to judge the reasonableness of answers, and to justify selected procedures (Burns, 1985).

Current mathematics instruction also has been criticized for being too abstract, presenting concepts and skills before many children are able to learn them meaningfully (Allardice &

Ginsburg, 1983, Baroody, 1989a, Ginsburg, 1989). When children do not understand what they are being taught, they often resort to rote memorization without developing understanding (Baroody, 1989a; Baroody, in press). Youngsters then fail to transfer procedures that they have learned to novel situations (Baroody, in press), or they apply procedures in an unthinking manner (Schoenfeld, 1982). Further, these students often come to conclude that school or formal mathematics involves nothing more than the memorization and mastery of procedures that have little relevance and meaning for real life problem solving (Schoenfeld, 1987).

Calls for Change

The education, business, scientific, and mathematics communities have expressed concern over the status of mathematics performance among American youth. It is believed that the level of mathematics performance among young people must increase if our country is to compete internationally in the scientific, technological, and business arenas. Thus, calls for change in how mathematics is taught abound. In 1989, the National Council of Teachers of Mathematics (NCTM) released a document titled, *Curriculum and Evaluation Standards for School Mathematics*, that recommends fundamental changes for how and what mathematics should be taught in elementary and secondary schools. The standards stress that students should (1) learn to value mathematics, (2) become confident in their ability to do mathematics, (3) become mathematical problem solvers, (4) learn to communicate mathematically, and (5) learn to reason mathematically. According to the standards, problem solving should be the focus of the mathematics curriculum, and mathematical principles and concepts as well as procedures should be taught. The importance of representations and illustrations in developing students' understanding of mathematical principles and concepts and the role of calculators and computers in freeing students from performing burdensome computations also are emphasized. Overall, the NCTM advocates a balanced instructional approach, one that includes the development of skills *and* conceptual understanding, of mathematical thinking and reasoning, and of problem-solving capabilities (National Council of Teachers of Mathematics [NCTM], 1989; Thompson & Rathmell, 1988).

The National Council of Teachers of Mathematics is not alone in its call for change. Other organizations such as the National Research Council (NRC) have joined NCTM in the critique of current mathematics instruction. The NRC's report, *Everybody Counts* (1989), urges a rethinking of the mathematics curriculum and how it is taught in our elementary and secondary schools.

"...approximately 80 percent of youngsters with learning disabilities...receive the dominant portion of their mathematics instruction in regular classrooms."

The national concern over the state of mathematical learning is understandable: traditional mathematics instruction is failing many students including those at risk (Carnine, in progress). But where do students in need of special education fit into the thinking about reform efforts? Many children with learning problems will inevitably be exposed to efforts to reshape mathematics education because approximately 80 percent of youngsters with learning disabilities and about 40 percent of students who are mildly retarded receive the dominant portion of their mathematics instruction in regular classrooms (Cawley et al., 1988). Thus, any changes made in the regular classroom involving curriculum, teaching methods, media and materials, and performance standards will affect numerous students with disabilities. Special educators quite naturally are arguing for reform efforts to be sensitive to the needs of students with learning problems.

The fostering of independent problem-solving skills that enable youngsters to apply mathematical procedures in functional, vocational, and career settings has been a long-standing goal in special education (Thornton, 1989a). Therefore, teaching methods and media and materials that have the potential for leading students toward this goal would be welcomed (Carnine, in progress; Carnine & Vandegrift, 1989, Cawley et al., 1988, Cawley & Miller, 1989, Thornton, 1989a).

Cognitive-based Mathematics. A Suggested Instructional Alternative

Cognitive-based methods for teaching mathematics are thought by their proponents to have the potential to lead both regular and special education students to a greater understanding of mathematical concepts and procedures. Cognitive-based approaches, which will be discussed in depth in Chapter Three, are founded on the beliefs that meaningful math learning requires the acquisition of conceptual as well as procedural knowledge and that students' independent problem-solving capabilities need to be nurtured.

This report presents a discussion of cognitive-based approaches to math instruction, their potential for use with students with disabilities, and their implications for media and material design and use. Topics addressed include the mathematical learning problems frequently observed among children with learning problems; the goals, principles, and research on which cognitive-oriented approaches are based; teaching methods and curricular emphases associated with cognitive-oriented instruction; and ways media and materials can be designed and used to support the teaching of mathematics from a cognitive perspective.

This publication is intended to provide publishers with a summary of the theories, principles, and research behind cognitive-based mathematics instruction and to focus on factors that have relevance for media and material design and use. It is hoped that the discussions contained herein will assist publishers to make informed, realistic decisions, thereby leading to more effective instruction for youth with learning problems.

CHAPTER TWO

Mathematical Learning Among Students with Disabilities--Problems and Potential

Historically more attention has been paid by instructional designers to the language arts deficiencies of students with learning problems than to their problems in mathematics courses (Blankenship, 1984; Fridriksson & Stewart, 1988). Yet studies from the classroom reveal that a substantial portion of youth with disabilities experience difficulties with mathematical learning. One survey revealed that 66.6% of students with learning disabilities at grade six and above were receiving special instruction in mathematics. Indeed, 26% of youngsters with learning disabilities were receiving special instruction *primarily* because of their mathematical deficiencies (McLeod & Armstrong, 1982).

Specific Areas of Mathematical Difficulties for Students with Learning Problems

The mathematical difficulties of students with learning disabilities range from those with basic mathematical computation to those with more advanced problem-solving activities. These youngsters tend to lack proficiency in basic number facts (Garnett & Fleischner, 1983, Goldman et al., 1988; Kirby & Becker, 1988; Thornton & Toohey, 1985); they often must stop and compute answers to number facts rather than directly retrieve answers from memory (Russell & Ginsburg, 1984).

Students who are mentally retarded also exhibit an array of difficulties with number facts learning and computation skills. And, as a rule, youngsters who are more severely retarded exhibit less mastery of crucial computation skills and concepts than do students who are mildly retarded (Baroody, 1986; Baroody & Snyder, 1983).

Not surprisingly, students with special learning needs, like their nondisabled peers, experience difficulties with word problem solving. While not the most sophisticated form of mathematical problems, word problems often require the application of more complex skills than do basic computational exercises. Young people need to understand the relationships presented in the problem and the actions to be carried out. Further, they need to be able to plan and execute a solution strategy (Riley et al., 1983).

Research indicates that students with learning disabilities have difficulties solving word problems, particularly those categorized as more difficult (Russell & Ginsburg, 1984), and Cruickshank (1948) determined that the greatest differences in mathematical performance between nondisabled average IQ students and their equivalent-mental-age peers who are retarded occurred in the area of verbal problem solving. Youngsters who are learning disabled and those who are retarded have particular difficulty with problems that contain extraneous information (Cawley, et. al., 1987; Cruickshank, 1948; Goodstein et al., 1971; Schenck, 1973). For example, when presented with a problem such as,

> There were 3 boys, 5 girls, and 2 dogs in the yard. How many children were in the yard?

students who are retarded often respond with an answer that represents the total of all numbers mentioned in the problem, such as 10 instead of 8 for the above example (Goodstein et al., 1971, Schenck, 1973). It has been suggested that a rote computation habit contributes to some of these errors (Goodstein et al., 1971).

The specific reasons for the difficulties with word problem solving among students with disabilities vary from child to child. An analysis of the difficulties with problem solving of students with learning disabilities conducted by Montague and Bos (in progress) determined that these youth have difficulties (1) predicting operations for solving problems, (2) selecting appropriate algorithms to solve multi-step problems, and (3) completing correctly problems after a decision is made about how to solve them. This research also determined that the mistakes of students with learning disabilities were not attributable to computational errors.

Not surprisingly the problem-solving performance of students in need of special education contrasts sharply with that of good problem solvers. The latter have an adequate, well-organized knowledge base (Pressley, 1986, Silver, 1987), are able to understand the nature of the problem to be solved (Silver, 1987), are capable of generating mental representations of the problem (Derry et al., 1987, Pellegrino & Goldman, 1987; Riley et al., 1983, Silver, 1987), and have knowledge of procedures and strategies that can be used to derive answers (Baroody, 1987, Montague, in press, Pressley, 1986). Moreover, good problem solvers possess metacognitive knowledge, i.e, knowledge that enables them to assess the demands of the problem, select and implement appropriate strategies, monitor the problem-solving process, and make modifications when selected strategies do not seem to work (Baroody, 1987, Garofalo & Lester, 1985, Montague, in press; Pressley, 1986, Silver, 1987).

Potential Capabilities of Students With Learning Problems

Are students with learning problems capable of becoming better problem solvers? Are they able to profit from instruction that stresses conceptual understanding? Can they acquire and appropriately apply an array of strategies while problem solving? In short, what evidence exists that students with disabilities would benefit from cognitive-based approaches to mathematics instruction?

A growing number of researchers are suggesting that the mathematical difficulties of many youngsters with learning disabilities are more characteristic of learning discrepancies or developmental delays than of developmental differences (Cawley, 1984b, Cawley et al., 1988, Goldman et al., 1988). In other words, students with learning disabilities often perform similarly to younger, nondisabled children on mathematical tasks (Garnett & Fleischner, 1983, Russell & Ginsburg, 1984), indicating that these youngsters have the capabilities to learn many of the mathematical ideas and procedures as their non-disabled peers, albeit at a slower rate.

"...the problem-solving performance of students in need of special education contrasts sharply with that of good problem solvers."

One area of research that has examined the capabilities of students in special education programs to become more effective learners and problem solvers involves the use of cognitive and metacognitive strategy instruction. Students with learning problems are frequently described as lacking in strategic knowledge (Scheid, 1989). When these youngsters do possess knowledge of strategies, they fail to apply it appropriately (Montague & Bos, in progress). Thus, strategy instruction is intended to help students acquire more efficient approaches to learning. Several strategy instruction projects have succeeded in teaching youngsters to do so (Scheid, 1989), and a few of these studies have been conducted within the area of mathematics. Some of these strategy instruction studies have aimed and succeeded at increasing the computational proficiencies of students in need of special education (Baroody, 1988b, Leon & Pepe, 1983, Lloyd et al., 1981, Schunk & Cox, 1986) and number facts learning (Baroody, 1988a; Thornton et al., 1983; Thornton & Toohey, 1985) of students in need of special education.

Word problem solving also has been addressed through strategy instruction research (Case & Harris, 1988; Fleischner et al., 1987; Montague & Bos, 1986). By teaching students problem-solving strategies, Case and Harris (1988) succeeded in improving the abilities of upper-elementary-level students with learning disabilities to solve one-step addition and subtraction word problems, and Fleischner and her colleagues (1987) assisted fifth and sixth grade youth with learning disabilities in learning how to solve four types of word problems: addition, subtraction, two-step problems and problems with extraneous information. Montague and Bos (1986) taught students with learning disabilities an eight-part process to apply to the solving of two-step word problems. Students were taught to (1) read the problem aloud; (2) paraphrase the problem aloud; (3) visualize the problem; (4) state the problem, i.e., what information is known and unknown; (5) hypothesize; (6) estimate; (7) calculate; and (8) self-check.

The goal of strategy instruction is to assist students to become independent learners. In mathematics instruction, that means equipping them with the knowledge and procedures that they can transfer to novel mathematical problems encountered in or out of school. Several of the above-cited studies attempted to measure if in fact students appropriately and independently applied instructional strategies following training (Case & Harris, 1988; Leon & Pepe, 1983; Montague & Bos, 1986, Schunk & Cox, 1986, Thornton & Toohey, 1985). As a rule, generalization did occur.

It should be noted that other populations of children with disabilities, including youngsters who are mentally retarded (Albion & Salzberg, 1982; Johnston et al., 1981; Leon & Pepe, 1983; Whitman & Johnston, 1983) and others who are severely behaviorally disordered (Davis & Hajicek, 1985), also have been successfully taught strategies to aid them in their mathematical learning and performance.

Implications for Instruction

Professionals have suggested that the mathematical difficulties experienced by students with learning problems may be largely due to or at least exacerbated by traditional curriculum and instruction (Baroody, 1987, Baroody, in press, Cawley et al., 1988; Fitzmaurice-Hayes, 1985a). If this is true, then more effective modes of instruction need to be sought. The results of the studies described in the preceding section support the position that students with special learning needs can be taught strategies for improving their mathematical performance.

It is true that effective strategy use represents only one aspect of mathematical thinking and performance as viewed from a cognitive perspective. Yet the research of cognitive and metacognitive learning strategies is encouraging because it underscores the *potential* of many students with disabilities to become more independent and thoughtful learners and provides evidence that these youngsters can be guided to more effective mathematics learning through methods other than those that have dominated their instruction, e.g., rote memorization and drill and practice (Case & Harris, 1988; Cawley, 1985b; Goodstein et al., 1971; Payne et al., 1981).

"...mathematical difficulties may be largely due to or at least exacerbated by traditional curriculum and instruction."

Why has drill and practice been the predominant form of mathematics instruction for students in need of special instruction? One possible explanation is that teachers believe youngsters who are disabled to be incapable of more meaningful mathematics learning or of engaging in problem-solving activities. Another reason may be that special education teachers, as well as many regular education teachers, feel inadequate to teach mathematics. A survey conducted in the mid 1980's found that nearly half of the responding resource teachers reported a lack of familiarity of different conceptual and theoretical approaches to mathematics (Carpenter, 1985). Fitzmaurice (1980), in an earlier survey of teachers of students with learning disabilities, noted similar results: nearly 71% of surveyed teachers so responded. Fitzmaurice's study also indicated that teachers lacked confidence in their abilities to teach a variety of areas of mathematics. For example, 50 percent stated that they lacked proficiency in teaching concepts involved in measurement, and 85.5 percent said they lacked competence to teach the metric system (Fitzmaurice, 1980).

The mastery of basic computation skills and knowledge of math facts is an important goal of mathematical learning for students in need of special education. But an increasing number of educators are challenging the wisdom of making these areas of mathematics learning the *only* or *most important* ones for learners with disabilities. Also being questioned is the indiscriminate use of or over reliance on drill and practice techniques. According to Hasselbring and his associates (1988), use of drill and practice alone is inappropriate and will result in little or no improvement in the math performance of students in need of special education. To maximize students' abilities to learn number facts, for example, attention needs to be paid to linking instruction to students' prior knowledge and to helping youngsters connect what they know through the building of declarative knowledge networks. That is to say, students need to be assisted in seeing the relationship among basic math problems such as 5 + 4 = 9 or 9 - 5 = 4 (Hasselbring et al., 1987). In general, teaching methods are being urged that will help students with learning problems to develop a greater understanding of mathematical concepts and their relationship to one another (Baroody & Snyder, 1983; Hasselbring et al., 1987) and acquire strategic and metacognitive capabilities (Thornton & Toohey, 1985).

The ultimate goal of mathematics instruction for students with learning problems is to assist them in acquiring the skills necessary to deal with the many unique mathematical problems that surface in everyday life (Cawley et al., 1988; Goodstein et al., 1971; Payne et al., 1981) Traditional mathematics instruction falls short of that goal, not just for students with learning problems but for many nondisabled youngsters as well. Carol Thornton (1989a) characterizes the prevailing mathematics curriculum as a deprived one, relying heavily on rote and contrived skill learning. What students with disabilities need to be exposed to, according to Thornton, is a language-based, active learning, developmentally appropriate, cognitive-based mathematics program that extensively utilizes applied problem solving.

Clearly, no one teaching method or approach is adequate for every student in every situation. But a growing number of special educators believe that cognitive-based approaches for mathematics instruction may better meet the need of students with learning problems than traditional approaches (Baroody, in press, Case & Harris, 1988, Cawley, 1985b; Cawley et al., 1988, Cawley & Goodman, 1969; Goodstein et al., 1971; Goodstein et al., 1972, Payne et al., 1981; Schenck, 1973). The reason for these beliefs as well as a discussion of the research and principles behind cognitive-based approaches for mathematics instruction appear in the next chapter.

CHAPTER THREE

Cognitive-based Principles for Teaching Mathematics

Foundation of Cognitive Beliefs

Cognitive-based instruction places prime importance on the development of youngsters' conceptual knowledge. It is believed that students must acquire an understanding of the concepts that underlie math procedures if they are to be successful problem solvers (Baroody & Ginsburg, 1986). Because of their emphasis on conceptual learning, cognitive-based teaching methods contrast sharply with traditional instructional approaches, which instead emphasize memorization of math facts and procedures. Cognitive theorists believe that the latter are not likely to lead many students, particularly those with learning problems, to a meaningful understanding of mathematics.

Besides stressing conceptual learning, cognitive-based theories are founded on the belief that children learn through constructing meaning rather than through an absorption-of-facts process. Children construct meaning by relating or assimilating new information with what they already know, by integrating previously isolated facts, or by adjusting existing knowledge to meet the demands of a new learning experience (Baroody, 1987, Baroody, 1989a, Baroody, in press). The next section provides an overview of some of the pertinent research findings pointed to by cognitive theorists in support of their beliefs.

Findings From Research on Children's Mathematical Thinking

A portrait of how youngsters' mathematical thinking develops has emerged from recent research on how young children acquire an understanding of basic mathematical processes. What are some of these pertinent research findings? **First, it is known that preschool-aged children *informally* acquire considerable math knowledge** (Allardice & Ginsburg, 1983; Baroody, 1987; Baroody & Ginsburg, 1986; Hiebert, 1984; Romberg & Carpenter, 1986). Informal mathematics is meaningful to children because it is developed through their own life experiences (Baroody, 1989a). According to Carpenter (1985), even before formal schooling, many children have reasonably sophisticated skills in solving word problems, attend to content, model problems, and invent effective procedures for computing. Preschool-aged children usually can count, and from their knowledge of counting they begin to understand several mathematical concepts such as same, different, and more (Baroody, 1987).

Second, while young children begin to understand many mathematical concepts and principles through their own experiences, they do so at different rates. It should not be assumed that all children at a given grade or age possess the same level of understanding. If instruction is provided in a uniform manner, some students will have a difficult if not an impossible time learning and assimilating the new information (Baroody, 1989a; Baroody & Ginsburg, 1986).

Third, research reveals that children progress through four levels of problem solving as they learn to effectively work addition and subtraction word problems (Carpenter & Moser, 1984). At the first level, children approach simple problems by modeling, i.e., objects are used and manipulated to represent and solve problems. At level two, students use both modeling *and* counting strategies. Level three marks the point at which children rely *primarily* on counting strategies, and at level four, children use math facts to answer questions (Carpenter, 1985). For example, children at the modeling stage will approach a problem such as

> Mike had 10 toy cars. He gave 3 to Kate. How many did he have left?

by taking 10 toy cars or other objects representing them and removing 3, then counting the remaining cars. Children who have progressed to counting strategies will count from 3 to the total or 10, while youngsters who have mastered basic math facts will directly retrieve the answer.

Children's abilities to use the most efficient strategy consistently is related to their developmental level. The gradual transition from one level to another involves significant advances in understanding and procedural skills (Carpenter, 1985; Carpenter & Moser, 1984).

Fourth, the degree of *success* students encounter when solving word problems depends not just upon their developmental level, but also upon the difficulty of the word problems encountered. Several factors contribute to word problem difficulty including the action required to solve the problem and the information that is and is not provided. Several taxonomies of word problems have been constructed by researchers (for example, see Carpenter, 1985, Peterson et al., 1988/1989; Riley, 1981, and Riley et al., 1983) to help illustrate differences among problem types and to provide guidance for teachers and instructional designers who develop and construct problems. Table One presents frequently referred-to categories of word problems. These examples illustrate how the complexity of problems change with the major action required (i.e, change, combine, compare, and equalize), the information that is provided, and the information that needs to be determined.

Studies have been conducted to determine how difficult these various types of problems are for young children to solve. Research has focused on problems categorized as *change*, *combine*, or *compare* items (Carpenter, 1985; Carpenter & Moser, 1982; Carpenter & Moser, 1984; Riley, 1981). Results of these studies indicate that generally most types of compare problems pose more difficulties for younger children (kindergartners and first graders) than do most type of problems in the change and combine categories (Riley, 1981). But it should be noted that considerable differences in difficulty are evident among items *within* categories. For example, combine problems that involve subtraction are more difficult for young children to solve than those involving addition, and change problems with the start unknown are more difficult than the other types of change problems (Riley, 1981).

As a rule, children gain proficiency in word problem solving within *all* categories as they progress through the primary grades, i.e., as they acquire more advanced concepts and skills (Carpenter, 1985, Carpenter & Moser, 1982; Carpenter & Moser, 1984, Riley, 1981, Riley et al., 1983), and it is believed that children can be assisted in their concept and skill development if their instruction incorporates an array of word problems that vary in their complexity (Fennema et al., in press).

"Children construct meaning by relating or assimilating new information with what they already know..."

The research findings summarized above have been largely ignored in practice. For example, typically, addition and subtraction instruction starts with modeling or teaching students to solve problems using concrete items. But then it proceeds *directly* to instruction of number facts mastery without taking into account that children use counting strategies after modeling and before fact use retrieval (Carpenter & Moser, 1984, Romberg & Carpenter, 1986). Word problems, when they are used in instruction, frequently are of the less challenging varieties such as those requiring change by adding or substraction with the results unknown (Peterson et al., 1988/1989).

TABLE ONE

Taxonomy of Word Problem Types[a]

CHANGE	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
by adding	Maria has 3 crayons. Kyle gave her 4 more. How many crayons does Maria have now?	Maria has 3 crayons. How many more does she need to have 7?	Maria had some crayons. Kyle gave her 3 more. Now she has 7. How many crayons did Maria have to start with?
by subtracting	Maria had 7 crayons. She gave 4 to Kyle. How many crayons does Maria have left?	Maria had 7 crayons. She gave some to Kyle. Maria has 3 crayons left. How many crayons did she give to Kyle?	Maria had some crayons. She gave 4 to Kyle. She has 3 left. How many crayons did Maria have to start with?

COMBINE	TOTAL MISSING	PART MISSING
by adding	Abby has 10 orange balloons and 2 green ones. How many balloons does she have altogether?	
by subtracting		Abby has 12 balloons. Two are green and the rest are orange. How many orange balloons does Abby have?

COMPARE	DIFFERENCE UNKNOWN	COMPARED QUALITY UNKNOWN	REFERENT UNKNOWN
by adding	Joey has 12 pencils. David has 7 pencils. How many more pencils does Joey have than David?	David has 7 pencils. Joey has 5 more pencils than David. How many pencils does Joey have?	Joey has 12 pencils. He has 5 more pencils than David. How many pencils does David have?
by subtracting	Joey has 12 pencils. David has 7 pencils. How many fewer pencils does David have than Joey?	Joey has 12 pencils. David has 5 fewer pencils than Joey. How many pencils does David have?	David has 7 pencils. He has 5 fewer pencils than Joey. How many pencils does Joey have?

EQUALIZE	DIFFERENCE UNKNOWN	COMPARED QUALITY UNKNOWN	REFERENT UNKNOWN
by adding	Jesse has 6 stickers. Tina has 4 stickers. How many more stickers does Jesse have than Tina?	Tina has 4 stickers. If she collects 2 more, she will have the same number of stickers as Jesse. How many stickers does Jesse have?	Jesse has 6 stickers. If Tina collects 2 more stickers she will have as many stickers as Jesse. How many stickers does Tina have?
by subtracting	Jesse has 6 stickers. Tina has 4 stickers. How many stickers does Jesse need to lose to have the same number of stickers as Tina?	Tina has 4 stickers. If Jesse loses 2 stickers he will have the same number of stickers as Tina. How many stickers does Jesse have?	Jesse has 6 stickers. If he loses 2 he will have the same number of stickers as Tina. How many stickers does Tina have?

[a]This taxonomy was constructed based on information appearing in Baroody, A., and Stanifer, D. (in progress); Carpenter (1985); Peterson, P., Fennema, E., and Carpenter, T. (1988/89); Riley, M. (1981); and Riley, M., Greene, J., and Heller, J. (1983).

Guiding Principles of Cognitive-based Instruction

What general instructional principles can be deduced from research on children's mathematical thinking?

Instruction should take into account children's developmental readiness. Instruction needs to be sensitive to how children mature cognitively (Fennema et al., in press), and it needs to be designed to facilitate the acquisition of concepts that lead to greater understanding (Baroody, in press; Fennema et al., in press; Fuson & Secada, 1986; Secada et al., 1983; Thornton et al., 1983; Thornton, 1989b). Learning proceeds from the concrete, incomplete, and unsystematic to the abstract, complete, and systematic. Students progress through these stages at different rates, and these variations in student learning patterns must be taken into account when planning instruction (Baroody, in press).

Instruction should link new information to existing knowledge. This principle, related to the first, stresses that math instruction should be built upon what students already know (Baroody, 1987). The informal skills and knowledge of mathematics that most children, including students who are disabled, possess can serve as the basis for more formal math learning (Baroody, 1987; Baroody & Ginsburg, 1984, Baroody & Ginsburg, 1986; Baroody, in press, Carpenter & Moser, 1984). Thus, the techniques, procedures, and symbols of formal mathematics should be explicitly linked to what children have learned informally (Hiebert, 1984). For example, the number sentence $5 + 5 = 10$ may seem strange to young children unfamiliar with mathematical symbolism. However, when a connection is drawn between this symbolism and counting done on fingers or with manipulatives, children begin to see the relationship between what they know informally and what they need to learn (Baroody, in press). For many students, including those with disabilities, learning problems can develop because formal mathematics is instructed outside the context of students' informal mathematical knowledge (Baroody, 1987; Baroody, 1989a; Hiebert, 1984; Resnick, 1987).

Instruction should emphasize the development of mathematical thinking. Reasoning, conceptual understanding, and recognizing patterns and relationships should all be goals of mathematics instruction (Baroody, in press; NCTM, 1989). Teaching mathematics within a problem-solving framework, where learned skills are applied to oral or written problems that have solutions not readily apparent, is believed to assist students to develop their mathematical thinking (Baroody, 1989a; Cawley & Miller, 1986; Fennema et al., in press; Peterson et al., 1988/1989; Thornton, 1989a).

Instruction should promote the learning of strategies. An emerging principle of cognitive-based approaches is the need to assist students to develop and appropriately use an array of cognitive and metacognitive learning strategies (Baroody, in press; Garofalo, 1987; Garofalo & Lester, 1985; Montage & Bos, 1986; Montague & Bos, in progress; Schoenfeld, 1987). Cawley and Miller (1986) point out that metacognitive skills related to planning, self-monitoring and self-evaluation are associated with good mathematical problem solving. Thus students should receive explicit instruction in how to develop these capabilities (Baroody, in press; Cawley et al., 1988; Cawley & Miller, 1986; Cherkes-Julkowski, 1985b; Garofalo & Lester, 1985; Montague & Bos, in progress; Schoenfeld, 1987).

Instruction should foster a positive disposition toward mathematics. Cognitive theorists acknowledge the role that attitudes, beliefs, and motivation play in the learning process. Instruction therefore should be designed to encourage motivation and positive beliefs (Baroody, in press; Holmes, 1985). Providing a supportive learning environment, helping students establish attainable learning goals, incorporating challenging and interesting problems in mathematics instruction, and stressing that effort affects achievement all enhance students' motivation (Holmes, 1985).

"...learning problems can develop because formal mathematics is instructed outside the context of students' informal mathematical knowledge."

Several instructional programs and approaches incorporating the above principles have been developed. Examples include the Cognitively Guided Instruction program developed by Thomas Carpenter and Elizabeth Fennema of the University of Wisconsin and Penelope Peterson of Michigan State University; the Verbal Problem Solving for Mildly Handicapped Project developed by John Cawley at the State University of New York at Buffalo;

the Mathematics Strategies Program, a component of the Strategies Intervention Model produced by the Institute for Research in Learning Disabilities at the University of Kansas and directed by Jean Schumaker and Donald Deshler; the Math Problem Solving Project directed by Marjorie Montague at the University of Miami; strategies for teaching math facts and computation formulated by Carol Thornton of Illinois State University and her associates; and the techniques espoused by Arthur Baroody of the University of Illinois for helping preschool and primary students develop their mathematical thinking. Contact information for these individuals is available in Appendix A.

While these programs and techniques illustrate the diversity of approaches that bear the label of cognitive-based mathematics instruction, they are all founded on the belief that many students with learning problems are capable of achieving a deeper understanding of mathematics when instruction is guided by cognitive-based principles. Not surprisingly several common characteristics and components of cognitive-based mathematics instruction have emerged from research and practice. These commonalities provide points for consideration and guidance to publishers contemplating the design and publication of materials that reflect a more cognitive-oriented approach to mathematics instruction for students with learning problems. The next chapter provides a discussion of these characteristics.

CHAPTER FOUR

Instructional Components of Cognitive-based Mathematics Teaching

In Chapter Three the underlying principles of cognitive-based mathematics instruction were identified and discussed. From research on and implementation of these programs a set of instructional features has emerged that could serve as guidelines not only for educators desiring to teach mathematics to from a cognitive perspective to youngsters with learning disabilities but also for developers and publishers wishing to produce resources that support teachers in doing so.

There is no question that teachers make or should make the key instructional decisions about what is taught in the classroom and how, but well designed student materials can greatly influence and support those decisions. Textbooks in particular play a powerful role in education since they are viewed by teachers as authorities on knowledge and as guides to teaching (Romberg & Carpenter, 1986). For many areas of the curriculum, including mathematics, how teachers approach a topic is guided by the content and organization of the textbook (Crosswhite, 1987; Trafton, 1984). Thus embedded in the instructional features discussed below are implications for how media and materials could be designed and used to support mathematics teaching from a cognitive perspective.

The instructional components discussed are grouped into those relating to content for instruction and those relating to the methods for teaching the content. The chapter ends with suggestions for how teacher guides accompanying student materials could be designed to provide further instructional support for teachers of cognitive-based approaches.

What Should Be Taught

Comprehensive Curriculum. Professionals advocating cognitive-based approaches to mathematics instruction for students with learning problems argue for a mathematics curriculum that goes beyond a focus on math facts and computation (Bley & Thornton, 1981; Bulgren & Montague, 1989; Cawley et al., 1988; Thornton et al., 1983). Calls for a more in-depth mathematics curriculum for students with disabilities are based on a belief that many of these youngsters can achieve beyond current levels *if* they are exposed to developmentally appropriate, meaningful instruction (Bulgren & Montague, 1989; Cawley, 1970; Cawley et al., 1988).

Cawley and his colleagues (1988) have proposed a "priority" curriculum that includes topics such as space, relations, and figures; basic operations with whole numbers; fractions; measurement; and problem solving. Other professionals have suggested that specific content strands be embedded in and integrated throughout the mathematics curriculum for students with learning problems. Estimation, functions, probability, statistics, algebraic reasoning, translation of symbols, logic, spatial reasoning, geometric figures and properties, and use of calculators have been suggested as strand topics (Bulgren & Montague, 1989). It is acknowledged, though, that not all students with disabilities will be able to master all the concepts involved in these areas (Cawley et al., 1988); indeed, some of these youngsters may not be able to progress beyond the most basic procedures and concepts.

Guidance for designing curriculum and instruction within some of the curricular areas referred to above is available in sources such as *Cognitive Strategies and Mathematics for the Learning Disabled* (1985), *Developmental Teaching of Mathematics for the Learning Disabled* (1984), and *Secondary School Mathematics for the Learning Disabled* (1985), all edited by John Cawley; *Mathematics for the Mildly Handicapped--A Guide to Curriculum and Instruction* (1988) by John Cawley, Anne Marie Fitzmaurice-Hayes, and Robert Shaw; *A Guide to Teaching Mathematics in the Primary Grades* (1989) and *Children's Mathematical Thinking* (1987) by Arthur J. Baroody; *Teaching Mathematics to Children with Special Needs* (1983) by Carol Thornton, Benny Tucker, John Dossey, and Edna Bazik; and *Teaching Mathematics to the Learning Disabled* (1981) by Nancy Bley and Carol Thornton. The National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards for School Mathematics* (1989) also provides examples of teaching ideas and activities. More information about these publications is contained in the Bibliography of this report.

Media and Materials Implications. Media and materials, particularly textbooks, could assist teachers of students with learning problems by providing an integrated presentation of topics across units and chapters. For example, a topic introduced in an earlier unit could be explicitly related to newly introduced topics, and activities could be contained throughout texts that would help reinforce and further develop skills introduced earlier (Bulgren & Montague, 1989). Too, materials could informally introduce topics through activities presented before the topic is *formally* taught.

Teachers of students in need of special education could be aided by textbooks that allow for the flexible presentation of content (Carnine & Vandegrift, 1989). Considerable variation in learning potential exists among and within categories of students with disabilities, but as a rule, these youngsters learn at a slower rate than nondisabled students (Callahan & MacMillan, 1981; Carnine & Vandegrift, 1989), and they will not be able to cover as much content as students without learning problems (Callahan & MacMillan, 1981, Carnine & Vandegrift, 1989). Teachers are helped when materials

- identify those areas and activities that are most important to emphasize and those which could be de-emphasized (Carnine & Vandegrift, 1989);
- present content in small steps (Bley & Thornton, 1981) and in a format that is clear and understandable (Callahan & MacMillan, 1981);
- provide meaningful reinforcement and further development of skills introduced earlier (Bulgren & Montague, 1989), and
- provide ample practice activities at the concrete and conceptual as well as the symbolic level (Bley & Thornton, 1981; Cawley, 1984c).

Concepts and Relationships. Mathematics instruction should emphasize conceptual understanding as well as procedural learning (Cherkes-Julkowski, 1985b; Fennema et al., in press; Fitzmaurice-Hayes, 1984). The thoughtful application of skills is only possible when concepts are understood. The National Council of Teachers of Mathematics (1989) describes concepts as the substance of mathematical knowledge, and Holmes (1985) defines them as ideas that represent a class of objects or events that have certain characteristics in common. Place value, one-half, square, rational number--are all examples of broad concepts.

"The thoughtful application of skills is only possible when concepts are understood."

Conceptual knowledge not only is necessary to understand the meaning behind mathematical procedures, but also for determining when those procedures are appropriate to apply in new situations. Too, emphasis on instruction of concepts may help prevent the development of misunderstandings or "bugs" that result in arithmetical errors (Resnick & Omanson, 1986).

Klausmeier and Ripple (1971) have provided some guidelines for how concepts should be taught. They suggest emphasizing the attributes of the concept, establishing the correct terminology for concepts, attributes, and instances; informing students of the nature of the concepts to be learned, providing for proper sequencing of the instances of concepts, encouraging and guiding student discovery, providing for the use of the concept; and encouraging independent evaluation of the attained concept.

Instruction should provide students with opportunities that will lead them to see how concepts apply in a variety of situations. Ample opportunities to generalize learned concepts should be provided to students with learning problems since these youngsters are known to have difficulties utilizing their knowledge in novel situations (Baroody, in press; Bley & Thornton, 1981; Deshler et al., 1981; Fitzmaurice-Hayes, 1985b).

Helping students see relationships also should be an instructional priority. Lesson content should be framed to draw connections between what a youngster already knows and understands and what is to be learned (Allardice & Ginsburg, 1983, Baroody, in press, Fennema et al., in press; Fridriksson & Stewart, 1988; Silver, 1987; Trafton, 1984). This instructional *connecting* needs to commence when formal mathematics instruction is first presented, since most students, including those with learning problems, start school with a store of informal mathematical knowledge upon which formal school instruction can be built (Baroody, 1987, Baroody, 1989a; Romberg & Carpenter, 1986).

Another goal of instruction should be helping students to see patterns and relationships among concepts (Baroody, 1989a, Baroody, in press, Fennema et al., in press, Hiebert, 1984, Holmes, 1985; Peterson et al., 1988/1989); between concepts and mathematical procedures (Baroody, in press, Hiebert, 1984); and between real world applications and school mathematics. As Fitzmaurice-Hayes (1985b) stresses, it is through the recognition of patterns and relationships that ideas about concepts and rules are initially formed. Furthermore, students should be shown how procedures can be represented symbolically and given the opportunity to make these connections, for example, by constructing number sentences to represent the problem posed in a verbal problem (Fennema et al., in press). Care also should be given to explicitly illustrating the connection between procedures with which children are familiar and the symbols that represent the procedures (Baroody, 1987; Cawley, 1989).

Media and Materials Implications. Materials can emphasize conceptual learning and mathematical relationships by providing ample illustrations and representations of concepts (Fitzmaurice-Hayes, 1985b). In particular, using a variety of examples of concepts as well as illustrations that do *not* represent the concept, i.e., non examples, such as is shown below for the concept of one-half, helps to foster concept development (Baroody, in press; NCTM, 1989).

Materials could include activities that actively involve children in making connections between mathematical ideas or concepts. According to Baroody (1989b), the learning of the concept of place value could be facilitated by use of worksheets picturing individual items, such as sticks, cars, stars, and so on, that children would be asked to group. Doing so helps youngsters to see the connection between individual units and groups of units, for example, that seven individual items or *units* can be placed into a *group* containing seven items. Such a method for teaching place value instruction contrasts with the usual presentations found in texts and other materials. Typically, students are shown pre-bundled items--ten sticks, for example--that are intended to represent a group of ten. According to Baroody, representations of pre-bundled items do not help children to actively construct the unit and group concepts (Baroody, 1989b).

Classroom resources also could contain illustrations that help students see the relationships between symbolic representations and the procedures for which they stand (Baroody, 1987; Cawley, 1989). The following is such an example.

#####	+	###	=	
5	+	3	=	

Patterns and relationship recognition also should be reinforced through materials. The following example, based upon an activity suggested by Fitzmaurice-Hayes (1985b), illustrates how students can be helped to see relationships:

Look at the shapes, then follow the directions below.

1. For each shape

- Find the sum of the angles.
- Divide the sum by 180.
- Compare your answer to the number of sides in the shape.

2. Compare your answers for each of the figures. Do you see a pattern?

Within materials, concept instruction should precede or accompany procedural instruction (Carnine & Vandegrift, 1989), and materials should never use explanations that are conceptually incorrect for the sake of expediency (Carnine & Vandegrift, 1989). For example, directions for completing long division problems sometimes instruct students to begin solving an item such as $5\overline{)127}$ by asking, "Does 5 go into 1?" and if the answer is no to then ask, "Does 5 go into 12?" This type of direction can lead to confusion since the 1 referred to is actually 100, and the 12 is 120. While children may be easily taught this procedure, it will do little to expand their understanding of what they actually are doing when they divide (Carnine & Vandegrift, 1989).

Strategy Learning. One of the goals of cognitive-based mathematics instruction is to help students to become more strategic learners (Baroody, in press; Goldman, 1989; Mayer, 1985, Thornton & Smith, 1988; Thornton & Wilmot, 1986). As illustrated in Chapter Two, many students with disabilities are thought capable of learning cognitive and metacognitive strategies to assist them in becoming more efficient, effective, and independent learners. General strategies that have been identified as contributing to effective mathematical problem solving are visualization and mental imagery, pictorial representation or diagram production, estimation, and checking one's progress (Montague, in press, Montague & Bos, in progress). And numerous strategies have been developed to assist students in performing specific procedures. For example, Thornton and Toohey (1985) have produced and tested strategies that help students master basic number facts.

Learning strategies can assist students to learn, but care must be given to teaching strategy instruction in a meaningful manner and within the context of conceptual learning discussed above. Strategies should *not* contribute to superficial understandings of mathematical procedures (Carnine, in progress). The "key word" approach is an oft-cited example of a strategy gone wrong (Baroody, in press; Cawley & Miller, 1986; Schoenfeld, 1982; Schoenfeld, 1988). Students are taught that "key" words in word problems signal certain operations, e.g., "more" signals the need for addition, as is the case in the illustration below.

> Joe had 4 marbles, Kyle gave him 3 *more*. How many marbles does Joe now have?

However, the following problem also uses the word "more," but solving it requires subtraction, not addition:

> Kate has 8 marbles. She has 2 *more* marbles than Jennifer. How many marbles does Jennifer have?

A student blindly applying the "key word" strategy would erroneously produce an answer of 10.

Thus, cognitive strategies must be taught thoughtfully (Baroody, in press). Students should be informed of the reason for learning and using a strategy and instructed about when it should and should not be used (Palincsar, 1986; Pressley, 1986). Too, students should be led to see how multiple strategies may be applied to solve problems (Peterson et al., 1988/1989).

Whether or not students thoughtfully and appropriately apply cognitive strategies is dependent in large measure upon youngsters' metacognitive capabilities (Cherkes-Julkowski, 1985b, Garofalo & Lester, 1985, Lester, 1985). Metacognitive learning behavior involves assessing the demands of a learning task and planning, implementing, monitoring, and evaluating the selected approach to accomplish the learning task. Problem solving requires that students possess not just an adequate content knowledge and knowledge of techniques for representing and translating problems, but also metacognitive processes for selecting and monitoring their implementation of solution strategies (Kilpatrick, 1985).

Students with learning problems are in particular need of instruction that will help them to develop their metacognitive capabilities (Cawley & Miller, 1986; Cherkes-Julkowski, 1985b; Fitzmaurice-Hayes, 1985b; Rivera & Smith, 1987; Thornton & Wilmot, 1986). Cherkes-Julkowski (1985b) offers a few instructional ideas for helping them do so. She suggests that students (1) be given a problem and asked to plan the steps to its solution, (2) be given answers to problems, then be required to determine the steps that were taken to solve them, and (3) be directed to talk out loud as they attempt to solve a problem.

Media and Materials Implications. Materials could assist students to learn and appropriately apply an array of cognitive and metacognitive strategies by providing demonstrations of the use of strategies, explanations of the purpose for and reasoning behind the application of the strategies, and illustrations of how strategies can be applied in a variety of settings. Materials also can provide exercises such as those requiring students to identify when the application of a specific strategy facilitates or works against the solving of certain problems.

Marginal notes or other prompts could be added to help students to stop and determine what is known in a problem, what needs to be known, and what strategies may be appropriate to apply. Students can be reminded to monitor their implementation of problem solutions, evaluate their answer, and reflect on the problem-solving process. Videotapes may be particularly helpful in illustrating these behaviors to students.

"...a problem-solving approach should be used to introduce youngsters to mathematical operations and the reasoning behind them."

Attitudes and Beliefs. Instruction should not ignore the need to develop positive beliefs and attitudes toward mathematics. Students with disabilities often have negative self concepts relating to their ability to learn in general and learn mathematics in particular. These perceptions may be accentuated by instruction that places too much emphasis on memorization of facts and procedures. Such instruction may contribute to the belief that mathematics is composed of a set of facts and procedures that are not related to real-world problems and situations (Baroody, 1989a, Schoenfeld, 1987). Too, an undo emphasis on speedy problem solving may lead students who are slower in mathematics performance to conclude that they are incapable of grasping mathematical ideas (Baroody, in press).

Students with learning problems need be explicitly taught that: it is smart to ask questions when they do not understand; errors are a natural part of learning; and mathematical knowledge gleaned from daily living experiences is relevant to understanding the formal mathematics taught in school (Baroody, in press). It is believed that instruction based upon cognitive principles by its nature helps to minimize the formation of negative attitudes and beliefs.

How Should Cognitive-based Math Be Taught?

Problem Solving. Presenting mathematics instruction within a problem solving context has been strongly recommended (Baroody, 1987; Bley & Thornton, 1981; Cawley, 1984a; Cawley & Miller, 1986; Fennell, 1983; Fennema et al., in press). It is believed that a problem-solving approach should be used to introduce youngsters to mathematical operations and the reasoning behind them (Baroody, 1987; Carnine & Vandegrift, 1989; Cawley, 1989; Peterson et al., 1988/1989). The following activity illustrates how children can be lead to an understanding of division through such an approach.

> Step One: Divide students into small groups. Give one child in each group several cups. Give a second student in each group two cups. Ask the first child to give the same number of cups as was given to the second student to every other child in the group.
>
> Step Two: Give one student in each group some cups and direct the child to distribute them so that each group member has the same number of cups.
>
> Step Three: Give one student in each group some cups and direct the student to *divide* the cups in such a way so that all students in the group have an equal number (Carnine & Vandegrift, 1989).

Through the approach described above, students are informally presented with the concept of dividing in the context of sharing--an issue that is important to children. Such problems allow students to work from their knowledge base and to become comfortable with the concepts before the word "division" and its formal, symbolic representation are introduced (Cawley, 1989).

Word problems, either written or posed orally, can also serve the purpose of engaging students in a problem-solving activity and helping them to improve their problem-solving capabilities (Peterson et al., 1988/1989; Fennema et al., in press). As was discussed in the last chapter, not all word problems are of equal difficulty or require the same strategies to be solved. Word problems used in instruction should be challenging enough to lead students to more sophisticated problem-solving behavior.

For example, materials developers are advised when producing, selecting or adapting items to:

- Use nonroutine problems. These include items that have too much, too little, or incorrect information; can be solved in more than one way; have multi-steps; have more than one possible answer; and/or require an analysis of the unknown (Baroody, 1987). Examples of some of these types of problems appear later in this section.

- Modify problems as necessary to accommodate the learning problems of students. For example, if a student has difficulty reading a problem, rewrite it (Cawley et al., 1987).

- Consider using a few interesting and challenging problems as opposed to many trivial ones (Baroody, in press; Bley & Thornton, 1981; Cawley, 1989).

- Allow students to construct their own word problems (Bulgren & Montague, 1989; Cawley et al., 1987;).

Problem-based approaches to teaching mathematics, then, should serve to extend students' conceptual knowledge (Holmes, 1985), provide youngsters with the opportunity to apply the procedures and skills they have acquired (Zhu & Simon, 1987), foster the development of metacognitive capabilities (Cawley et al., 1987), and illustrate why and how mathematics is important in daily living.

Media and Materials Implications. Media and materials can play a major role in helping teachers to foster the problem-solving capabilities of special education students. For example,

- Materials could feature word problems as vehicles for introducing mathematical procedures as opposed to using them solely as end-of-lesson practice exercises (Baroody, 1987, Baroody, 1989b, Cawley et al., 1987, Cawley, 1989; Peterson et al., 1988/1989).

- A variety of word problems could be incorporated into instructional resources (Baroody, 1987; Carnine, in progress; Cawley et al., 1987; Marten, 1989). Textbooks in particular have been criticized for the preponderance of simple word problems included as exercises (Carnine, in progress). One analysis of elementary math textbook series revealed that over 90% of the word problems could be solved by applying the "key word" strategy referred to earlier (Cawley, 1985b; Cawley et al., 1988). Particularly helpful in encouraging thoughtful problem-solving are nonroutine word problems (Baroody, 1987). Examples of these types of problems follow:

Analysis of the unknown:
Max and Steve want to buy a Frisbee that costs $4.00. Max has $1.00 and Steve has $2.00. Do Max and Steve have enough money to buy the Frisbee?

Too much, too little, or incorrect information:
Ann ate 2 brownies for dessert. Her brother Peter ate 1. There are 6 brownies left. How many brownies did both Ann and Peter eat?

Leslie gave 2 baseball cards to Jill, 4 to Keith, and 3 to Brian. How many baseball cards does Leslie have left?

Problems solved in more than one way:
Anna had 50 cents when she went to the grocery store. She wanted to buy a candy bar that cost 40 cents and a jawbreaker that cost 5 cents. Did she have money enough to buy both? (This problem can be solved by adding the cost of the items and subtracting that from 50 cents or by subtracting 40 cents from 50 cents then subtracting 5 cents from 10 cents).

Multi-step problem:
Tim has painted 5 pictures to give away as presents. He wants to give 1 each to his mother, his father, his grandmother, his grandfather, his uncle, his sister, and his brother. Has he painted enough pictures?

Problems with more than one answer:
Julie is at her school festival. She has 90 cents. Balloons cost 25 cents, candied apples cost 35 cents, hot dogs cost 50 cents and ride tickets cost 25 cents each. What can Julie buy?

▸ Materials could include problems that extend over time, integrate mathematics with other subjects, require the application of a variety of math procedures, necessitate the collection and analysis of data, and require the drawing of conclusions (Bulgren & Montague, 1989; Carnine & Vandegrift, 1989, Cawley, 1989). The materials could also include suggestions for altering the complexity of such problems to match the ability level of the targeted students (Carnine & Vandegrift, 1989).

Extended problem-solving activity emphasizes the utility of mathematics in everyday life and illustrates that many problems require solving over time. For example, the Verbal Problem Solving for the Mildly Handicapped Project developed by John Cawley includes problem-solving units that require students to apply a variety of mathematical processes over time. For example, one such unit requires students to measure plant growth. Students plant seeds and identify conditions related to plant growth that they wish to evaluate. During the course of the unit students make a variety of measurements at given intervals to assess the height and breadth of the plants, and they chart the results of these measurements. Students then evaluate their observations and draw conclusions. Thus, a student taking part in this unit has an opportunity to perform a variety of mathematical functions including measuring, computing, recording data, and graphing (Cawley, 1989).

▸ Problems should be utilized that are based on situations and topics that are of interest to students and/or relate to their world (Bley & Thornton, 1981; Bulgren & Montague, 1989; Callahan & MacMillan, 1981; Cawley et al., 1988; Cawley et al., 1987). Familiar contexts allow students to utilize their prior knowledge in interpreting the demands of the problem, and high interest contexts obviously promote motivation.

Questioning and Listening. Teachers presenting cognitive-based instruction need to rely heavily on questioning and listening to students (Garofalo & Standifer, 1989). Teachers can use information obtained from a student's explanation of his or her reasoning and thought processes to assess and analyze the student's degree of understanding (Fennema et al., in press; Garofalo, 1987; Good et al., 1983). To engage in questioning and listening, particularly of individual students, requires that instruction be organized to allow teachers the opportunity to interact with students. One method that helps facilitate this interaction is small group instruction.

Small Group Instruction. Research indicates that small group work can enhance students' conceptual development and computational capabilities (Slavin et al., 1984; Slavin & Karweit, 1985). Small group work also is believed to facilitate problem solving (Garofalo & Standifer, 1989; Holmes, 1985; Schoenfeld, 1987; Silver, 1985). Group work necessitates communication and discussion among members about the problem to be solved. Talking about problems can help youngsters to integrate new knowledge with what they already know (Fitzmaurice-Hayes, 1985b; Thornton, 1989a), and justifying their selection of solution approaches and listening to their peers do so can lead students to more mature problem-solving strategies (Fennema et al., in press).

Media and Materials Implications. Materials, particularly textbooks, could provide more activities specifically designed for group problem solving. Too, such group problem-solving activities offer opportunities to embed mathematical-related problems within the context of other subject areas such as science, social studies and health. Good and his colleagues (1989/1990) point out that the lack of curriculum materials designed for small group work has served to impede implementation of this method of instruction in mathematics. These authors also point out that when materials are lacking and teachers must create their own classroom resources for group work, lack of continuity of content within classes and across grades often results. Well designed media and materials could help provide such continuity.

Modeling. Teacher modeling of problem-solving activities and strategy applications is a technique frequently used in teaching to demonstrate procedures or cognitive strategies for solving problems, to explain the reasoning behind the actions and to demonstrate metacognitive behavior (Cherkes-Julkowski, 1985b; Garofalo, 1987; Henderson, 1986; Herrmann, 1989; Lloyd & Keller, 1989; Schoenfeld, 1987; Schunk, 1981; Silver, 1987). Modeling has the potential for being an effective instructional technique when it does *not* lead students to the false conclusion that mathematical problem solving is a neat, clear cut process (Schoenfeld, 1987). Cherkes-Julkowski (1985b) warns that many students are adept at memorizing and performing steps to a process modeled by the teacher without having grasped the meaning behind it. As with other techniques, teachers need to use modeling judiciously and in combination with other methods such as questioning and listening.

Manipulatives. Use of manipulatives is frequently recommended as a good method for providing a concrete visualization of abstract concepts and of actively involving students in the learning process (Cawley, 1989; Fleischner et al., 1982; Good et al., 1983; Hendricks, 1983; Holmes, 1985; Kennedy, 1986; Thornton & Wilmot, 1986). However, although manipulatives can accomplish these ends, they do not *automatically* provide support for abstract thinking (Baroody, 1989c, Callahan & MacMillan, 1981; Garofalo & Standifer, 1989). That is to say, students can mindlessly manipulate items without reflecting on the *why* of their activity or without understanding the reasoning behind it (Baroody, 1989c).

"Successful use of manipulatives requires thoughtful planning and organization."

Successful use of manipulatives requires thoughtful planning and organization (Martin & Carnahan, 1989). Thornton and Toohey (1986) offer guidelines for using manipulatives with students in need of special education. They suggested that the teacher question students about their actions as they work with manipulatives; have students verbalize their thinking; require students write out the problems that they have solved with manipulatives, and have students use manipulatives to check answers.

Media and Materials Implications. Publishers of manipulatives should include guidelines for how these items could be employed to teach concepts and procedures. Textbooks could provide directions and recommendations for when manipulatives could or should be used in the illustration of a concept or procedure.

Calculators. Many educators believe that greater use of calculators would free students from burdensome calculations and give them more time to engage in problem-solving activities (Bulgren & Montague, 1989; Callahan & MacMillan, 1981, Cawley & Miller, 1986; Fitzmaurice-Hayes, 1985c; NCTM, 1989). However, calculators should not be used as a substitute for procedural understanding. Fitzmaurice-Hayes (1985c) cautions that knowledge of basic number concepts, understanding of place value, knowing the four operations, and some knowledge of mathematical facts should be prerequisites for calculator use. Too, introduction of calculators into instruction underscores the need to teach students to estimate and judge the reasonableness of their answers (NCTM, 1989).

It is important to remember that calculator use does not come naturally to many students and that some students will need to be explicitly instructed and given practice in the appropriate and effective application of calculators (Bulgren & Montague, 1989).

Media and Materials Implications. Materials should provide explicit instruction in the application of calculators in problem solving and incorporate exercises and problems that guide students to greater proficiency. Activities that provide students with practice in estimating and judging the reasonableness of answers should be interwoven throughout materials (Bulgren & Montague, 1989).

Teacher Guides

The teacher guides that accompany student materials also could provide invaluable support to teachers. Some specific recommendations for information that should appear in the teacher guide includes the following:

Information About Children's Mathematical Development. Cognitive approaches stress the need for teachers to be sensitive to children's mathematical development. Many teachers are not aware of the research that describes the normal course of growth in children's mathematical thinking and how instruction can facilitate or hinder students' mathematics learning. Clear summaries of this research and its implications for instruction of specific concepts and procedures should be included in teacher guides (Bulgren & Montague, 1989; Garofalo & Standifer, 1989).

Instructional Suggestions. Teachers should be provided with numerous ideas for how to approach the teaching of mathematical concepts, strategies, and procedures (Bulgren & Montague, 1989). These suggestions should help teachers to introduce a lesson, listen to and question students, prompt students' prior knowledge, and present the lesson. Videotapes illustrating the application of the various techniques suggested would be particularly helpful (Garofalo & Standifer, 1989). Sample scripts may also be of assistance to many teachers (Baker, 1989; Carnine & Vandegrift, 1989).

Instructional Adaptations. Cognitive-based instruction stresses the importance of adapting instruction to meet the learning needs of students. This is particularly important to do when teaching students with disabilities. Materials could assist teachers by providing examples of how activities could be adapted to make them more accessible to some students, e.g., making problems less complex by substituting smaller for larger numbers (Carnine & Vandegrift, 1989, Cawley et al., 1987) and by suggesting alternative algorithms (Bley, 1989, Carnine & Vandegrift, 1989, Cawley, 1984c).

Goal Coordination. Teachers new to cognitive-based methods for mathematics instruction may be concerned that such methods will not address the teaching of traditional skills, a particularly acute concern when the district has established performance objectives that must be met for students to be promoted or graduated. Hence, charts or matrices that list traditional skills along with how, when and where they are addressed in the materials should be included in the teacher guide (Bulgren & Montague, 1989).

Assessment Suggestions. Materials should include guidelines and mechanisms to help teachers to ascertain students' level of understanding before, during and after instruction (Carnine & Vandegrift, 1989, Cawley, 1984c; Garofalo & Standifer, 1989). While ongoing, informal assessment is an integral part of cognitive-based instruction, formal assessments also are important. But the latter should include more than paper and pencil, multiple choice tests (Carnine & Vandegrift, 1989). Teachers need to be provided with techniques and ideas for designing assessment processes that will help them determine the degree to which youngsters understand and apply math concepts and procedures.

Teachers would also be helped by the inclusion of guidelines for analyzing common computational errors made by students (Carnine & Vandegrift, 1989; Maurer, 1987). Children frequently develop "buggy" algorithms due to misunderstandings of concepts. Teachers can be shown how to identify these bugs and given suggestions for leading students to an understanding of the concepts and correct procedures.

Summary

It is hoped that the above suggestions provide some guidance to materials developers and publishers for how media and materials may support the teaching of mathematics from a cognitive perspective. Once again it needs to be stressed that the role of media and materials in cognitive-based education is secondary to the role of the teacher. Media and materials alone cannot or should not be the primary force in instruction. Yet well-designed classroom resources can support teachers in their efforts and in many instances may be the way teachers are introduced to cognitive theories. Publishers and developers are advised that the Information Center for Special Education Media and Materials maintains a database of media and materials that are useful in the instruction of children with disabilities. Media and materials have been identified that reflect a cognitive-based perspective for teaching mathematics, and while the Center does not evaluate the adequacy of these items, it does collect descriptive information intended to assist educators in locating appropriate classroom resources. Examples of database records are contained in Appendix B.

CHAPTER FIVE

Conclusions and Considerations

Regular and special educators are becoming increasingly interested in cognitive-based methods for mathematics instruction. There is a growing belief that traditional methods of mathematics instruction need be modified if American youngsters are to reach their potential in mathematics learning. Cognitive-based mathematics instruction, viewed as an alternative to current educational practices, is supported in part by results of research of children's mathematical thinking. These studies substantiate the contention that young children move gradually from concrete to abstract modes of problem solving in addition and subtraction. This progression occurs as children acquire an understanding of the meaning of concepts and procedures. Cognitive theorist believe that this understanding can be aided but not forced by instruction.

While much is known about the progression of young children's mathematical thinking, less is known about how mathematical understanding develops beyond the primary grades. Research is needed to determine how older students acquire an understanding of more complex concepts. Future research also should aim to ascertain the appropriate balance between direct, active teaching of mathematical topics and guided, independent learning that surrounds problem-solving approaches (Thornton, 1989a), study the ways mathematical concepts and skills should be sequenced to maximize learning and prevent the formation of misconceptions (Thornton, 1989a); and determine what are the most important instructional variables in leading students to become independent, strategic problem solvers (Montague & Bos, in progress).

Media and Material Design Decisions

The research base supporting cognitive-oriented approaches to mathematics instruction is best classified as a developing one. Yet, from the information that is available, several suggestions relating to media and material design and use have been forthcoming and were the subject of the preceding chapter of this report. Publishers must necessarily address issues about the feasibility and profitability of making the changes suggested.

Some of the foregoing recommendations such as those calling for the inclusion of more nonroutine word problems, extended problem-solving activities, and small group learning tasks in materials could be addressed by publishers with relative ease. Other suggested modifications would require more extensive rethinking and reworking of media and materials. This is particularly true of the recommendations involving the design of textbooks to meet the learning needs of students with disabilities, the sequencing of activities to reinforce previously introduced topics, the increased emphasis on teaching concepts and identifying relationships and patterns, and the framing of instruction in a problem-solving context. Publishers desiring to make these changes should seek the advice and consultation of professionals with varying expertise: special educators, mathematics educators, and mathematicians.

Publishers may be concerned that incorporating all of the suggested changes would require the production of lengthy, consumer intimidating products. Indeed, producing more comprehensive, interrelated, problem-solving-oriented materials that are sensitive to the needs of students with learning problems would require many publishers to rethink product format as well as content. For example, decisions need to be made about whether a single text, a series of texts per grade, or supplemental items targeted to specific groups of students would be the best format for such materials.

Recommendations for enhancing the teacher guides to classroom resources by, for example, including alternative modes for presenting content to youngsters with learning problems, summaries of pertinent research information such as that explaining how children think about mathematics, and assessment guidelines for use to ascertain students' conceptual understanding also would require more extensive materials as well as a rethinking of the role of teacher guides. The recommendations discussed in the last chapter suggest that guides should emphasize the why of the instructional approach as well as the how. Would teacher guides so designed be sufficient sources of "inservice" information? Would teachers with no prior knowledge of cognitive-based methods be able to make use of the research information provided? Should other items such as videotapes be produced by publishers to illustrate the research principles and methods addressed in materials?

Is There a Market for Cognitive-based Mathematics Materials?

Of [illegible]urse the major concern for publishe[illegible] whether a market exists for materia[illegible]luced to facilitate cognitive-based mathematics instruction. It is fair to assume that relatively few experienced teachers, particularly teachers of students with learning problems, currently employ cognitive-based teaching methods in their instruction. Few of these teachers were introduced to cognitive-oriented theories during their professional education program, but other factors also contribute to the limited use of cognitive-based methods with youngster who are disabled. Some special education teachers believe that students with learning problems are incapable of the type of mathematical thinking and reasoning used in problem-solving type activities and needed for the understanding of mathematical concepts. Too, teachers in a school district that places a high priority on standardized test results or that requires students to meet specific performance objectives that stress skill knowledge will understandably construct lessons that reflect district priorities.

"...calls for change will surely provoke discussion and examination of current teaching practices..."

While all the above are negatives from a market perspective, other factors point to a growing interest in cognitive-based methods and a concomitant demand for materials that facilitate teaching of these methods. Dissatisfaction with mathematical performance of American youth has resulted in calls for change. Instructional modifications advocated by groups such as the National Council of Teachers of Mathematics and the National Research Council stress teaching from a problem-solving perspective, emphasizing conceptual learning, building upon students' prior knowledge, and designing instruction sensitive to how youngsters think about and learn mathematics. These recommendations mirror many of the principles of cognitive-based methods. While school-based educators will not follow the lead of professional groups in a lock-step manner, these calls for change will surely provoke discussion and examination of current teaching practices within school systems and colleges of education throughout the country. Numerous articles relating to cognitive-oriented approaches to mathematics education and the instructional recommendations from professional groups have appeared in practitioner-oriented publications such as *Educational Leadership*, *Arithmetic Teacher*, and *Instructor*.

Second, special educators are beginning to examine some of their past assumptions about their students' capabilities in the area of mathematics learning. Increased attention is being paid to helping these youngsters to become more effective problem solvers. Evidence of this heightened attention is reflected in several articles that have appeared in periodicals targeted to special education professionals such as *Learning Disability Focus*, *Journal of Learning Disabilities*, *Focus on Exceptional Children*, *Journal of Reading, Writing, and Learning Disabilities*, and *Teaching Exceptional Children*.

Third, the influential role on curriculum of standardized and other tests that place heavy emphasis on skill learning is being questioned. A growing number of professionals are arguing that skill testing as reflected in traditional standardized tests should constitute only one measure of students' mathematical performance. More emphasis should be placed on assessing students' understanding and problem-solving capabilities.

Finally, as research continues and programs and projects incorporating research findings are developed and implemented, increasing numbers of prospective teachers will be exposed to cognitive theories during their professional training.

While all these trends are promising, the fact remains that teachers of students with learning problems as a rule are not currently familiar with or practitioners of cognitive-based methods for teaching mathematics. It is not clear whether teachers unfamiliar with these methods and the theories behind them would make effective use of materials designed from a cognitive perspective. It is likely that for many teachers, such materials would actually serve as their introduction to cognitive-based principles.

Cognitive-based mathematics instruction is a developing area. As such many questions remain as to how that instruction can best be implemented and how media and materials can best support teachers. Available evidence does underscore the potential of cognitive-based methods for educating students with disabilities. But for that potential to be actualized requires teamwork: professional education programs need to teach university students and experienced teachers the theories and principles behind this form of mathematics instruction; special education and regular education teachers need to thoughtfully utilize this knowledge to enhance students' problem solving capabilities; and developers and publishers need to supply the materials to support teachers in their efforts.

APPENDIX A

1989 Instructional Methods Forum Participants

Janice Baker
University of Pittsburgh
5N15 Forbes Quadrangle
Pittsburgh, PA 15260
412-648-7192

Ms. Baker is the site coordinator in Pittsburgh for the Arithmetical Verbal Problem Project. In that capacity, she works with teachers who are field testing the materials developed for use in the project. Ms. Baker also serves as Co-director for Project MELD, through which technical assistance is provided to school districts for mainstreaming learning disabled students, and an effective model for full-time mainstreaming of learning disabled elementary students is demonstrated.

Arthur J. Baroody, Ph.D.
College of Education
University of Illinois
1310 South Sixth Street
Champaign, IL 61820
217-333-8138

Dr. Baroody is an educational psychologist who is interested in children's mathematical development. His research focuses on the learning of counting, numbers, arithmetic, and place-value skills and concepts. Dr. Baroody has written numerous articles and three books on teaching mathematics meaningfully to children: *Children's Mathematical Thinking*, *A Guide to Teaching Mathematics in the Primary Grades*, and *Elementary Mathematics Activities: Teachers' Guidebook.*

Nancy Bley
Park Century School
2040 Stoner Avenue
Los Angeles, CA 90025
213-478-5065

Ms. Bley has been at the Park Century School, a school for children with learning disabilities, since 1976. Initially a math specialist, she now serves as academic coordinator and is in charge of supervising the curriculum and the teaching staff. Ms. Bley is the coauthor with Carol Thornton of *Teaching Mathematics to Children with Learning Disabilities*, second edition. She also has written articles that have appeared in *Arithmetic Teacher* and *Teaching and Computers*.

Janis Bulgren, Ph.D.

Institute for Research in Learning Disabilities
223 Carruth-O'Leary
University of Kansas
Lawrence, KS 66045
913-964-4780

Dr. Bulgren is currently serving as the Project Director for the University of Kansas Institute for Research in Learning Disabilities' federally funded grant, Math Strategy Interventions for Learning Disabled Youth, and as Project Coordinator of the Development and Validation of Learning and Teaching Strategies for the Kansas City INROADS Pre-Collegiate Program. Dr. Bulgren was the recipient of the Council for Learning Disabilities' Award for Outstanding Research in Learning Disabilities in 1987.

Douglas Carnine, Ph.D.

University of Oregon
1751 Calder Street
Eugene, OR 97403
503-485-1163

Dr. Carnine is the author of numerous articles that focus on issues related to the effective design of instruction for special education students. He is the coauthor, along with Silbert and Stein, of *Direct Instruction Mathematics*, second edition. Dr. Carnine's major research interests include methods for developing automaticity and problem-solving capabilities in students with learning problems, and the role of technology in the education of special needs students.

Lisa Pericola Case

Prince George's County
9501 Greenbelt Road
Lanham, MD 20706
301-459-7566

Ms. Case is a special education teacher in the Prince George's County, Maryland, school system. She has conducted research on the use of self-instructional strategy training to improve the math problem-solving abilities of learning disabled students. Ms. Case currently teaches orthopedically impaired youngsters and has an interest in exploring how to modify materials for the physically handicapped.

John Cawley, Ph.D.

State University of New York at Buffalo
593 Baldy Hall
Amherst, NY 14051
716-636-3174

Dr. Cawley's major work has been in mathematics instruction for learning disabled students. In recent years he has served as editor of such books as *Cognitive Strategies and Mathematics for the Learning Disabled*, *Developmental Teaching of Mathematics for the Learning Disabled*, and *Secondary School Mathematics for the Learning Disabled*. He has co-written, along with Anne Marie Fitzmaurice-Hayes and Robert Shaw, the book, *Mathematics for the*

Mildly Handicapped. Dr. Cawley's current research interests are verbal problem solving among the handicapped, randomized sequencing of computation processes with handicapped, and the role of regular classroom teachers as primary instructional sources for special education students.

Laura Cohn
9212 Ida Lane
Morton Grove, IL 60053
312-966-9822

Ms. Cohn is a student and research assistant working with Dr. Arthur Baroody at the University of Illinois. She has worked on projects that have studied students' addition and multiplication, and has coauthored with Dr. Baroody an article about the math performance of a learning disabled student.

Lacey Cooper
Open Court Publishing
407 South Dearborn, Suite 600
Chicago, IL 60605
312-939-1500

Mr. Cooper is the Vice President in charge of mathematics at Open Court Publishing. Open Court's Real Math textbook series includes a major emphasis on thinking skills, problem-solving strategies, and applications.

Cathleen Deery
Syracuse City Schools
429 Tompkins Street
Syracuse, NY 13204
315-422-1578

Ms. Deery is currently teaching an integrated program for students with autism and non-labelled students in the Syracuse School System. In this capacity she teaches regular curriculum, adapting academics, and functional self-care, and community living skills. She also serves as a consultant teacher for those labelled students who are placed in regular education classrooms. She recently presented a paper on integrated classrooms to the Association of Persons with Severe Handicaps.

Sharon Derry, Ph.D.
Department of Psychology
Florida State University
Tallahassee, FL 32306
904-644-3075

Dr. Derry serves as Director of Cognitive and Behavioral Sciences in the Psychology Department of Florida State University. She has authored several articles related to mathematical problem solving and cognitive strategy research. Her current research interests include cognitive theories of problem solving, learning strategies, computer-assisted instruction, intelligent tutoring systems, human tutorial interaction, word problems, and everyday problem solving.

Elizabeth Fennema, Ph.D.
University of Wisconsin/Madison
225 North Mills
Madison, WI 53706
608-263-4265

Dr. Fennema has been an elementary school teacher, an educator of teachers at both the preservice and inservice level, and a researcher. Her two main research interests are gender differences in mathematics and applying cognitive and instructional science research findings to changing the elementary school mathematics curriculum. She is the developer, along with Thomas Carpenter and Penelope Peterson, of the Cognitively Guided Instruction, an approach to learning mathematics with understanding.

Anne Marie Fitzmaurice-Hayes, Ph.D.
College of Basic Studies
University of Hartford
Bloomfield Avenue
West Hartford, CT 06117
203-243-4931

Dr. Fitzmaurice-Hayes teaches mathematics to college students with a history of difficulty in the subject. She is the author, along with John Cawley and Robert Shaw, of *Mathematics for the Mildly Handicapped*. Dr. Fitzmaurice-Hayes' current research interests are effective rehearsal strategies for the college student who has both a limited background in mathematics and a severe mathematics phobia, and female mathematicians of the past and present.

Jeannette E. Fleischner, Ph.D.
Department of Special Education
Teachers College
Columbia University
New York, NY 10027
212-678-3860

Dr. Fleischner is a teacher educator and serves as Director of the Child Study Center at Teachers College, Columbia. Her professional interests include assessment, instructional planning, remedial teaching of handicapped students, and math learning disabilities. Dr. Fleischner has authored several publications that explore the issue of mathematics learning among students with handicaps.

Nancy Fones, Ph.D.
Scholastic Inc.
730 Broadway
New York, NY 10003
703-338-3007

Dr. Fones serves as the Director of Training and Sales/Marketing Support for the Software Division of Scholastic Inc. She is in charge of all training and coordinates the sales efforts of Scholastic sales representatives and Scholastic's authorized education dealers. Prior to her work in publishing, Dr. Fones was a member of the faculty at the Model Secondary School for the Deaf at Gallaudet College.

Joe Garofalo, Ph.D.
University of Virginia
Ruffner Hall
Charlottesville, VA 22903
804-924-0845

Dr. Garofalo is on the faculty of mathematics education at the University of Virginia at Charlottesville. He has written several articles that focus on the role of metacognition in mathematics learning. He is the editor, along with Frank Lester, of *Mathematical Problem Solving: Issues in Research*. Dr. Garofalo has a general research interest in problem solving. Currently he is analyzing data from a project that explored the problem solving strategies used by seventh graders.

Karen R. Harris, Ed.D.
Department of Special Education
University of Maryland
College Park, MD 20740
301-454-2118

Dr. Harris has been involved in a series of studies validating self-instructional strategy training among mildly to moderately handicapped learners. She has authored several articles about self-instructional strategy training. Her current research focuses on strategy training in the areas of general problem solving, written language, and mathematical problem solving.

James Hargest
Harford County Schools
45 East Gordon Street
Bel Air, MD 21014
301-838-7300

Mr. Hargest, along with Dr. Carolyn Wood, Supervisor of Research, Testing, and Evaluation for Harford County Schools, and other district staff members, contributed to the development of several curricular guides, one of which is *A Learning Strategies Approach to Functional Mathematics for Students with Special Needs.*

Beth Ann Herrmann, Ph.D.
University of South Carolina
203 Wardlaw
Columbia, SC 29208
803-777-4836

Dr. Herrmann's research interests are cognitive strategy instruction, cognitive assessment techniques, staff development, teacher metacognitive control of instruction, and effective instruction at the teacher education level. She has conducted reading and mathematics studies of the use of the Direct Explanation model of instruction and a series of studies focusing on the development of teachers' knowledge structures as well as the interrelationships between teachers' knowledge structures and their instructional practices.

Mazie Jenkins

Marquette Elementary School
1501 Jenifer Street
Madison, WI 53703
608-267-4242

Ms. Jenkins has taught primary level mathematics in the Madison, Wisconsin, public schools for fifteen years. She has served on a variety of district committees, including the Minority Students Achievement and Whole Language Committees. She is currently teaching inservice classes on Black Children's Literature and the Cognitively Guided Instruction (CGI) mathematics education program. Ms. Jenkins has been a CGI teacher for three years and has coordinated the pilot CGI program at the Marquette Elementary School.

Clayton Keller, Ph.D.

Department of Child and Family Development
120 Montague Hall
University of Minnesota/Duluth
Duluth, MN 55812-2496

Dr. Keller taught behavior disordered students for eight years prior to pursuing graduate work in special education. He recently coauthored a chapter on cognitive training implications for arithmetic instruction and an article on effective mathematics instruction. Dr. Keller's current research interests are the areas of learning disabilities in math, subtypes of learning-disabled students, uses of computer technology for the disabled, and persons with disabilities as teachers.

Maris Manheimer

Montgomery County Public Schools
850 Hungerford Drive, Room 226
Rockville, MD 22055
301-279-3384

Ms. Manheimer is currently serving as an educational diagnostician in the Montgomery County, Maryland, school system. She has served as a secondary level resource teacher. Ms. Manheimer has been involved in curricular development efforts and has conducted inservice in the areas of the assessment of special education students and learning strategies instruction.

Barbara J. Marten

Madison Metropolitan School District
214 Green Lake Pass
Madison, WI 53705
608-267-4282

Ms. Marten has spent most of her professional career as a primary school teacher in Madison, Wisconsin. Currently she teaches in the Open Primary, a class for children in first and second grades, which provides each child with a sequentially planned program for the development of cognitive, language, thinking, learning, social, and basic skills, as well as learning strategies. Special education students are mainstreamed into the Open Primary. Ms. Marten has participated in the Cognitively Guided Instruction mathematics education project for three years.

Cecil D. Mercer, Ph.D.
Multidisciplinary Diagnostic and Training Program
University of Florida
2806 N.W. 29th Street
Gainesville, FL 32605
904-392-0702

Dr. Mercer is the author of several articles and books addressing the instruction of special education students. Examples of the latter include *Teaching Students with Learning Problems*, with A.R. Mercer, and *Students with Learning Disabilities*. His current research interests include the number of trials to master math facts, teaching exceptional students to apply mathematical concepts, and the effectiveness of low-stress algorithms.

Marjorie Montague, Ph.D.
School of Education
University of Miami
P.O. Box 248065
Coral Gables, FL 33124
305-284-2891

Dr. Montague teaches special education at the University of Miami. Her research interests focus on cognitive and metacognitive strategies for improving mathematical problem-solving and composition skills for students with learning disabilities, particularly students at the middle-school level. She is the author of several articles that discuss the problem-solving capabilities of learning disabled students and describe interventions for helping improve these students' performances.

Sheridan Osterstrom
Buffalo Board of Education
50 Heward Street
Buffalo, NY 14207
716-875-2532

Ms. Osterstrom is a special education teacher in Buffalo Public Schools. She has taught mentally retarded and learning disabled students in both self-contained and resource room settings. She currently is working as the on-site coordinator of the Verbal Problem-Solving among the Mildly Handicapped project, directed by Dr. John Cawley. In this capacity she is responsible for, among other things, staff inservice. Her professional interests include how to better prepare teach .s to teach effectively, and better prepare students to learn.

Janet Pittock
Creative Publications
788 Palomar Avenue
Sunnyvale, CA 94086
408-720-1400

Ms. Pittock taught grades three to five for five years prior to her involvement in publishing. Her responsibilities at Creative Publications include working with their product development team, conducting workshops on materials usage, conducting market surveys, and producing product catalogs.

M. Lewis Putnam, Ph.D.
Department of Exceptional Student Education
Florida Atlantic University
Boca Raton, FL 33431-0991
407-367-3280

Dr. Putnam's research interests are primarily in the area of academic and social interventions for adolescents at risk of school failure. He served as coordinator for a project designed to develop learning strategies in the area of mathematics for mildly handicapped adolescents while at the Institute for Research in Learning Disabilities at the University of Kansas. Currently he is developing procedures for effectively mainstreaming handicapped students into regula classrooms.

Diane M. Rivera, Ph.D.
Dept of Exceptional Student Education
Florida Atlantic University
P.O. Box 3091
Boca Raton, FL 33431-0991
407-397-3280

Dr. Rivera served as the District Coordinator of Special Education Staff Development for the Albuquerque Public Schools. In that role, she coordinated all staff development activities. Dr. Rivera has written articles related to mathematics education, including those that address the topic of the use of strategy instruction to teach basic mathematic skills. Generalization training is one of her current research interests. Currently she is on the faculty of Florida Atlantic University's College of Education.

Dale Seymour
Dale Seymour Publications
P.O. Box 10888
Palo Alto, CA 94303
415-324-2800

Mr. Seymour is the president of Dale Seymour Publications. This firm publishes an array of mathematical materials, including items for teaching problem solving to low math achievers, and a variety of manipulatives. Prior to entering publishing, Mr. Seymour held a variety of teaching and administrative positions in the public schools. He served on the Commission on Standards for School Mathematics of the National Council of Teachers of Mathematics and has authored or coauthored over 60 mathematics education publications.

Dorothy Standifer
Paxton Community Schools
520 North Tenth Avenue
Hoopeston, IL 60942
217-283-6568

Ms. Standifer, a primary classroom teacher and Chapter I mathematics instructor for the past twenty years, teaches in Paxton Community Schools in Illinois. Ms. Standifer is currently pursuing a doctoral degree in elementary mathematics and cognitive development, and she serves as a teaching assistant at the University of Illinois.

Linda J. Stevens
Pennsylvania Resources and Information Center for Special Education
200 Anderson Road
King of Prussia, PA 19406
215-265-7321

Ms. Stevens coordinates the production of a Pennsylvania statewide newsletter, the "PRISE Reporter," which reaches 17,000 special educators. Future issues of the newsletter will focus on alternate means of assessment, curricular coordination between regular and special education, and student support teams. She is also responsible for selecting and describing the interventions used by regular education teachers in a federally funded project at the University of Minnesota, "Student Learning in Context: A Model for Educating All Students in General Education Settings."

John F. Thomson
Educational Teaching Aids
2745 Oakview Drive
Rochester, NY 14617
716-342-9905

Mr. Thomson is the manager of the eastern region for Educational Teaching Aids (ETA). In this capacity, he conducts workshops for teachers in the use of manipulatives to support the teaching of math, writes and edits materials, is involved with product development and evaluation, and manages sales and consulting for ETA in the northeastern section of the country. Prior to his involvement with ETA, Mr. Thomson served as a mathematics teacher at the secondary level and as a mathematics coordinator for Title I.

Carol A. Thornton, Ph.D.
Department of Mathematics
Illinois State University
313 Stevenson Hall
Normal, IL 61761
309-438-8781

Dr. Thornton teaches mathematics education courses, directs a math learning clinic for children, and co-directs an NSF-funded undergraduate middle school teacher preparation project. She has authored 48 articles and 30 books, and has coauthored *Teaching Mathematics to the Learning Disabled* with Nancy Bley and *Teaching Mathematics to Children with Special Needs* with Tucker, Dossey, and Bazik. Among Dr. Thornton's current research interests are teaching and learning strategies for basic facts.

Judy Vandegrift
Addison-Wesley Publishing Company
2725 Sand Hill Road
Menlo Park, CA 94205
415-854-0300

Ms. Vandegrift has worked in the area of textbook publishing for over ten years. Currently serving as the Managing Editor of Elementary Mathematics at Addison-Wesley, she oversees the production of the elementary math text series and conducts training sessions. Prior to her involvement in publishing, Ms. Vandegrift was a teacher, serving as a mathematics specialist at the elementary level.

Carolyn Wood, Ph.D.
Harford County Schools
45 East Gordon Street
Bel Air, MD 21014
301-838-7300

Dr. Wood is the Supervisor of Research, Testing, and Evaluation with Harford County Schools in Maryland. Along with Jim Hargest and other district staff members, she contributed to the development of several curricular guides, one of which is *A Learning Strategies Approach to Functional Mathematics for Students with Special Needs.*

Guests

Thomas Berger
Instructional Materials Development Program
National Science Foundation

Edward Gickling, Ph.D.
Assistant Executive Director for Professional Development
Council for Exceptional Children

Sara Hines
Director, Tutoring
Lab School of Washington

Noel Kerns
Academic Supervisor
Lab School of Washington

Genevieve Knight, Ph.D.
Maryland Center for Thinking Studies
Coppin State College

Ethelianda Nelson
Academy of Mount Saint Ursula

Sidney S. Spindel
Teacher
Montgomery County Public Schools

Janice Welborn
Center for Systems in Program Development

U.S. Department of Education
Office of Special Education Programs Staff

Beatrice F. Birman, Chief
Research and Development Projects Branch
Division of Innovation and Development

Doris Cargile
Education Program Specialist

Martin Kaufman, Director
Division of Innovation and Development

Information Center for Special Education Media and Materials Staff

Victor Fuchs
Director

Charles Lynd
Information Specialist

Carol Bianchini Daniels
Associate Director

Karen Scheid
Research Specialist

APPENDIX B

Sample Records from the ICSEMM Database

-TITLE- THE UNIVERSITY OF CHICAGO SCHOOL MATHEMATICS PROJECT (UCSMP) SECONDARY COMPONENT MATERIALS (1990)
-AUTHOR- Zalman Usiskin, Project Director, and Flanders, Hynes Polonsky, Porter, Viktora, McConnell, Brown, Eddins, Hackworth, Sachs, Woodward, Hirschorn
-FORMAT- print curriculum. series of six student books, each can be accompanied by calculator; supplemental components for each book include: teacher's edition, teacher's resource file of blackline masters with storage crate, visual aids, solutions manual, software packages (available for Apple or IBM)
-COST- books and components priced separately, contact publisher representatives for costs
-GRADE- 7,8,9,10,11,12
-INTEREST- junior high, secondary
-DESCRIPTION- This is an instructional curriculum to teach mathematics with an emphasis on mathematical sciences, real world content/situations, critical thinking skills, use of calculators and computers. It is designed for students of average abilities at the intermediate and secondary level, the Transition Mathematics program, which prepares students for first-year algebra, can be started with gifted or high-achieving students in grade six or with remedial or low-achieving students in grade nine.

Multidimensional approach organizes material according to four main types of understanding or SPUR objectives: Skills (step-by-step procedures used to get answers), Properties (underlying mathematical principles), Uses (applications of mathematics in real situations), and Representations (graphs or pictures that show math concepts). Lessons incorporate questions, applications, review, and extension sections to promote comprehension and independent thinking. Self-tests with solutions enable students to self monitor progress.

Series titles and topics covered are. Transition Mathematics (applied arithmetic, pre-algebra, and pre-geometry with emphasis on real world applications); Algebra (four operations, applications, statistics, probability, geometry), Geometry (traditional, coordinate, and transformation approaches with applications and development of proof); Advanced Algebra (algebraic expressions and forms in real world applications, applied geometry, emphasis on graphing); Functions, Statistics, and Trigonometry with Computers--available 1991 (display, describe, transform and interpret numerical information in data, graph, and equation formats), Precalculus and Discrete Mathematics--available 1991 (integration with algebraic skills, emphasis on high-order mathematical thinking).

Teacher's edition features reduced student text pages with annotations, pre-chapter overview, objectives, teaching notes, follow-up activities, review material. Teacher's resource file includes over 600 blackline masters in five books including: Quiz and Test Masters, Lesson Masters, Computer Masters, Answer Masters, Teaching Aids (patterns for manipulatives, charts, graphs).

-APPROACH- learning strategies: mathematics, applied arithmetic; multidimensional

-EFFECTIVENESS- Background: This series was developed at the University of Chicago as the result of extensive research and consultation with a national advisory board of distinguished professors. Authors for the series were selected based on teaching experience and mathematics expertise. This program was the first full mathematics curriculum developed to implement the recommendations of the NCTM Standards committee. This program seeks to incorporate substantial changes to math curriculum including increased use of technology (calculators and computers), earlier introduction of higher order math concepts (algebra), and recommendation that mathematics be taught by mathematics teachers in the elementary grades. Zalman Usiskin, UCSMP Project Director, states "UCSMP is committed to technology because we believe students should be taught to do problems as adults do them and not be asked to go through torturous work simply because there is a long way to get an answer. In the real world, solutions arise from a variety of methods. Mental work is used. Estimation can be found at all stages of the solution process. Addition doesn't occur only in the addition chapter in a textbook. Algebra doesn't just occur in algebra " This statement is from the edited transcript of Usiskin's presentation entitled "The Beliefs Underlying UCSMP," which is available from Everyday MathTools Publishing Co., 1007 Church St., Suite 306, Evanston, IL 60201; (708) 866-0702.

Field test: Extensively field tested nationwide over several years with thousands of students Pilot tests were conducted by the initial team of authors. Further evaluation and revisions were based on national studies. For additional information on testing and evaluation, contact publisher at (800) 554-4411. Publisher states that "students using Transition Mathematics significantly outperformed comparison students in geometry and algebra readiness and also became effective calculator users without diminishing their arithmetic skills."

-PUBLISHER- Scott, Foresman and Company

-ADDRESS- 1900 East Lake Avenue
Glenview, IL 60025
(800) 554-4411
(708) 729-3000

-END-

-TITLE- THINKING STORY BOOKS
-AUTHOR- Stephen S. Willoughby, Carl Bereiter, Peter Hilton, Joseph H. Rubinstein, basal series authors
-FORMAT- print components: one set of 3 read-aloud books at primary level and one set of 3 student books and 3 teacher's editions at intermediate level
-COST- $23.50, each primary teacher read-aloud book; $4.10, each intermediate student book; $7.00, each intermediate teacher's edition
-READING- 1.0, 2.0, 3.0, 4.0, 5.0, 6.0
-GRADE- pre-, Ki, 1,2,3,4,5,6
-INTEREST- primary, elementary
-DESCRIPTION- This is an instructional series of supplemental components to teach mathematics with an emphasis on cognitive strategies, thinking skills, problem solving, and cooperative learning. It is designed as a set of interactive classroom materials for teachers and students, including whole class lessons and small group work. These materials are featured components of REAL MATH, a complete basal math program, which reflects the most recent NCTM standards. These materials were developed for students at primary and intermediate levels. They are suitable for use with students of diverse ability levels in mainstream classrooms, students with learning disabilities (LD), students with remedial math needs, or slightly older students who are mildly handicapped.

The primary level materials are: How Deep Is The Water (Grade 1), Measuring Bowser (Grade 2)-Spanish edition available, Bargains Galore (Grade 3). Primary level thinking stories are brief accounts of characters dealing with mathematics in real life situations. Questions which require students to employ math concepts, math facts, and math computation skills are integrated within each story. Implementation of these materials involves the teacher reading the story aloud to the class and pausing to ask questions as they appear in the text. These questions are open-ended and so promote thought processes in advanced, average, and slower learners. The class discusses information provided, determines appropriate operations, evaluates whether an answer is logical or absurd, and identifies which data are relevant to the questions asked. A set of word problems follows each short story, these problems emphasize thinking skills rather than drill and practice.

The intermediate materials are: Land, Iron and Gold, The Treasure of Mugg Island. Each student book features three complete stories that emphasize thinking skills and problem solving. These books and the problem solving activities contained are recommended for students to use in small cooperative learning groups. Suggestions for whole class, small group, and individualized activities are included in the teacher's editions.
-APPROACH- learning strategies: mathematics; cognitive-based; problem solving, thinking skills, cooperative learning
-EFFECTIVENESS- From publisher brochure "How Open's Court's REAL MATH helps you teach basic math skills": "In Real Math, we have tried to follow the recommendation of the National Council of Teachers of Mathematics (NCTM) and of other groups that computational practice not be limited to paper and pencil drill."

Field test: The Center for the Improvement of Mathematics Education evaluated the field testing of Real Math and conducted an independent Learner Verification Study. The field testing operation was monitored and evaluated under the direction of Leonard M. Warren, Executive Director, Center for the Improvement of Mathematics Education, San Diego, CA. Dr. Robert P. Dilworth, Professor of Mathematics, California Institute of Technology, Pasadena, directed the objective testing program and analyzed the results. A copy of the field test results and the complete Learner Verification Report are available by contacting the publisher at (800) 435-6850.
-PUBLISHER- Open Court Publishing Company
-ADDRESS- 407 South Dearborn
Chicago, IL 60605
(800) 435-6850
(800) 892-6831 (in IL)
(815) 223-2520 (in Alaska and Hawaii)
-END-

-TITLE- THINKER MATH: DEVELOPING NUMBER SENSE & ARITHMETIC SKILLS
-AUTHOR- Carole Greenes, Linda Schulman, and Rika Spungin
-FORMAT- Print: series of three 96-page 8 1/2" x 11 binders, each with 80 reproducible activity pages, organized at three grade levels (3-4, 5-6, 7-8)
-COST- $43.00, complete series; $16.75, each binder
-GRADE- 3,4,5,6,7,8
-INTEREST- elementary, junior high
-DESCRIPTION- This is an instructional series to teach mathematics and analytical reading with an emphasis on critical thinking skills, and problem solving. It is designed to be used as a supplement to any regular or special mathematics education program. Activities are highly recommended for classroom or small group discussions of problem solving strategies.

Each activity page consists of four short stories with important numbers extracted and placed in a display area on that page. Students apply reasoning, estimation, and logical thinking to restore the numbers in a fill-in-the-blank format so that the story makes sense mathematically and contextually.

Teacher guidelines, discussion suggestions, solutions and demonstration stories are included.

-APPROACH- Learning strategies: mathematics; thinking skills; problem solving skills
-EFFECTIVENESS- Field nominated: Contact: Carol Thorton, Department of Mathematics, Illinois State University, 300 Orlando Avenue, Normal, IL 61761; (309) 438-8781.
-PUBLISHER- Creative Publications
-ADDRESS- 5040 West 11th Street
Oak Lawn, IL 60453
(800) 624-0822 (Orders)
(800) 435-5843 (in IL)
(408) 720-1400 (Editorial/Marketing Offices)

-END-

BIBLIOGRAPHY

Albion, F. & Salzberg, C. (1982). The effect of self-instructions on the rate of correct addition problems with mentally retarded children. *Education and Treatment of Children, 5*, 121-131.

Allardice, B. & Ginsburg, H. (1983). Children's psychological difficulties in mathematics. In H. Ginsburg (Ed.), *The Development of Mathematical Thinking* (pp. 319-350). New York: Academic Press.

Baker, J. (1989, June). *Verbal Problem Solving for Mildly Handicapped Students in the Pittsburgh Public Schools.* Presentation at the ICSEMM Instructional Methods Forum, Washington, D.C.

Baker, L. & Brown, A. (1984). Cognitive monitoring in reading. In J. Flood (Ed.), *Understanding Reading Comprehension* (pp. 21-44). Newark, DE: International Reading Association.

Baroody, A. (1985). Basic counting principles used by mentally retarded children. *Journal of Research in Mathematics Education, 17*, 382-389.

Baroody, A. (1986). Counting ability of moderately and mildly handicapped children. *Education and Training of the Mentally Retarded, 21*, 289-300.

Baroody, A. (1987). *Children's Mathematical Thinking--A Developmental Framework for Preschool, Primary, and Special Education Teachers.* New York: Teachers College Press.

Baroody, A. (1988a). Mental-addition development of children classified as mentally handicapped. *Educational Studies in Mathematics, 19*, 369-388.

Baroody, A. (1988b). Number comparison learning by children classified as mentally retarded. *American Journal of Mental Retardation, 92*, 461-471.

Baroody, A. (1989a). *A Guide to Teaching Mathematics in the Primary Grades.* Boston: Allyn and Bacon.

Baroody, A. (1989b, June). *Designing Instruction to Foster Meaningful Learning and Problem-Solving Ability.* Presentation at the ICSEMM Publishers Workshop, Washington, D.C.

Baroody, A. (1989c). One point of view: manipulatives don't come with guarantees. *Arithmetic Teacher, 37*(October), 4-5.

Baroody, A. (in press). Teaching mathematics developmentally to children classified as learning disabled. In D. Reid, W. Hresko, and H.L. Swanson (Eds.) *A Cognitive Approach to Learning Disabilities.* Austin, TX: Pro-ED.

Baroody, A. & Ginsburg, H. (1984, April). *TMR and EMR Children's Ability to Learn Counting Skills and Principles.* Paper presented at the annual meeting of American Educational Research Association, New Orleans. (ED 249 681).

Baroody, A. & Ginsburg, H. (1986). The relationship between initial meaningful and mechanical knowledge of arithmetic. In J. Hiebert (Ed.), *Conceptual and Procedural Knowledge: The Case of Mathematics* (pp. 75-112). Hillsdale, NJ.: Erlbaum.

Baroody, A. & Snyder, P. (1983). A cognitive analysis of basic arithmetic abilities of TMR children. *Education and Training of the Mentally Retarded, 18*, 253-259.

Baroody, A. & Standifer, D. (in progress). Addition and subtraction. In R. Jensen (Ed.), *Early Childhood* volume of the National Council of Teachers of Mathematics Research Interpretation Project. New York: MacMillan.

Blankenship, C. (1984). Curriculum and instruction: An examination of models in special and regular education. In J. Cawley (Ed.), *Developmental Teaching of Mathematics for the Learning Disabled* (pp. 29-53). Rockville, MD: Aspen.

Bley, N. (1987). Problem solving and learning disabilities. *Arithmetic Teacher, 34*(April), 35.

Bley, N. (1989, June). *Topics and Issues.* Presentation at the ICSEMM Instructional Methods Forum, Washington, D.C.

Bley, N. & Thornton, C. (1981). *Teaching Mathematics to the Learning Disabled.* Rockville, MD: Aspen.

Borkowski, J., Estrada, M., Milstead, M., & Hale, C. (1989). General problem solving skills: Relations between metacognition and strategic processing. *Learning Disabilities Quarterly, 12*, 57-70.

Brown, A., Bransford, J., Ferrara, R., & Campione, J. (1983). Learning, remembering, and understanding. In J. Flavell & E. Markman (Eds.), *Handbook of Child Psychology, Vol. 3: Cognitive Development* (pp. 77-166). New York: John Wiley.

Brown, C., Carpenter, T., Kouba, V., Lindquist, M., Silver, E., & Swafford, J. (1988a). Secondary school results for the fourth NAEP mathematics assessment: Discrete mathematics, data organization and interpretation, measurement, number and operations. *Mathematics Teacher, 81*(April), 241-248.

Brown, C., Carpenter, T., Kouba, V., Lindquist, M., Silver, E., & Swafford, J. (1988b). Secondary school results for the fourth NAEP mathematics assessment: Algebra, geometry, mathematical methods and attitudes. *Mathematics Teacher, 81*(May), 337-347, 397.

Bulgren, J. & Montague, M. (1989, June). *Report from Working Group Four.* Presentation at the ICSEMM Instructional Methods Forum, Washington, D.C.

Burns, M. (1985). The role of questioning. *Arithmetic Teacher, 32*(February), 14-17.

Bush, W. (1987). Mathematics textbooks in teacher education. *School Science and Mathematics, 87*(November), 558-564.

Callahan, L. & MacMillan, D. (1981). Teaching mathematics to slow learning and mentally retarded children. In *The Mathematical Education of Exceptional Children and Youth* (pp. 146-190). Reston, VA: National Council of Teachers of Mathematics.

Carnine, D. (in progress). At risk students: Making the math connection.

Carnine, D. & Vandegrift, J. (1989, June). *Report from Working Group Three.* Presentation at the ICSEMM Instructional Methods Forum, Washington, D.C.

Carpenter, R. (1985). Mathematics instruction in resource rooms. Instructional time and teacher competence. *Learning Disabilities Quarterly, 8*, 95-100.

Carpenter, T. (1985). Learning to add and subtract: An exercise in problem solving. In E. Silver (Ed.), *Teaching and Learning Mathematical Problem Solving: Multiple Research Perspectives* (pp. 17-40). Hillsdale, NJ: Erlbaum.

Carpenter, T., Fennema, E., Peterson, P., & Carey, D. (1988a). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for Research in Mathematics Education, 19*, 385-401.

Carpenter, T., Fennema, E., Peterson, P., Chiang, C., & Loef, M. (1988b). *Using Knowledge of Children's Mathematics Thinking in Classroom Teaching: An Experimental Study*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans. (ED 292 983).

Carpenter, T. & Moser, J. (1982). The development of addition and subtraction problem solving skills. In T. Carpenter, J. Moser, & T. Romberg (Eds.), *Addition and Subtraction: A Cognitive Perspective* (pp. 9-24). Hillsdale, NJ: Erlbaum.

Carpenter, T. & Moser, J. (1984). The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education, 15*, 179-202.

Case, L. & Harris, K. (1988). *Self-Instructional Strategy Training: Improving the Mathematical Problem Solving Skills of Learning Disabled Students*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans.

Cawley, J. (1970). Teaching arithmetic to mentally handicapped children. *Focus on Exceptional Children, 2*(4), 1-8.

Cawley, J. (1984a). An integrative approach to needs of learning disabled children: Expanded use of mathematics. In J. Cawley (Ed.) *Developmental Teaching of Mathematics for the Learning Disabled* (pp. 81-94). Rockville, MD: Aspen.

Cawley, J. (1984b). Learning disabilities: Issues and alternatives. In J. Cawley (Ed.), *Developmental Teaching of Mathematics for the Learning Disabled* (pp. 1-28). Rockville, MD: Aspen.

Cawley, J. (1984c). Selection, adaptation, and development of curricular and instructional materials. In J. Cawley (Ed.), *Developmental Teaching of Mathematics for the Learning Disabled* (pp. 227-252). Rockville, MD: Aspen.

Cawley, J. (1985a). Cognition and the learning disabled. In J. Cawley (Ed.), *Cognitive Strategies and Mathematics of the Learning Disabled* (pp. 1-32). Rockville, MD: Aspen.

Cawley, J. (1985b). Thinking. In J. Cawley (Ed.), *Cognitive Strategies and Mathematics for the Learning Disabled* (pp. 139-161). Rockville, MD: Aspen.

Cawley, J. (1989, June). *Qualitative Enhancement of Mathematics Performance Among the Mildly Handicapped*. Presentation at the ICSEMM Instructional Methods Forum, Washington, D.C.

Cawley, J., Fitzmaurice-Hayes, A., & Shaw, R. (1988). *Mathematics for the Mildly Handicapped--A Guide to Curriculum and Instruction*. Boston: Allyn Bacon.

Cawley, J. & Goodman, J. (1969). Arithmetical problem solving: A demonstration with the mentally handicapped. *Exceptional Children, 36*, 83-90.

Cawley, J. & Miller, J. (1986). Selected views on metacognition, arithmetic problem solving, and learning disabilities. *Learning Disabilities Focus, 2*(1), 36-48.

Cawley, J. & Miller, J. (1989). Cross-sectional comparisons of the mathematical performance of children with learning disabilities: Are we on the right track toward comprehensive programming? *Journal of Learning Disabilities, 22*, 250-254, 259.

Cawley, J., Miller, J., & School, B. (1987). A brief inquiry of arithmetic word-problem-solving among learning disabled secondary students. *Learning Disabilities Focus, 2*(2), 87-93.

Cawley, J. & Vitello, S. (1972). Model for arithmetical programming for handicapped children. *Exceptional Children, 39*, 101-111.

CGI math teachers tune in to students' thinking. (1989). *WCER Highlights, 1*(1), 1-3.

Cherkes-Julkowski, M. (1985a). Information processing: A cognitive view. In J. Cawley (Ed.), *Cognitive Strategies and Mathematics for the Learning Disabled* (pp. 117-138). Rockville, MD: Aspen.

Cherkes-Julkowski, M. (1985b). Metacognitive considerations in mathematics instruction for the learning disabled. In J. Cawley (Ed.), *Cognitive Strategies and Mathematics for the Learning Disabled* (pp. 99-116). Rockville, MD: Aspen.

Cobb, P. (1988). The tension between theories of learning and instruc'' n in mathematics education. *Educational Psychologists, 23*, 87-103.
Crosswhite, J. (1987). Cognitive science and mathematics education: A mathematics educator's perspective. In A. Schoenfeld (Ed.), *Cognitive Science and Mathematics Education* (pp. 265-277). Hillsdale, NJ: Erlbaum.
Cruickshank, W. (1948). Arithmetic ability of men'ally retarded children, II. *Journal of Educational Research, 42*, 279-288.
Davis, R. & Hajicek, J. (1985). Effects of self-instructional training and strategy training on a mathematics task with severely behaviorally disordered students. *Behavioral Disorders, 10*, 275-282.
De Corte, E. & Vershaffel, L. (1981). Children's solution processes in elementary arithmetic problems: Analysis and improvement. *Journal of Educational Psychology, 73*, 765-779.
Derry, S., Hawkes, L., & Tsai, C. (1987). A theory for remediating problem-solving skills of older children and adults. *Educational Psychologist, 22*, 55-87.
Deshler, D., Alley, G., Warner, M., & Schumaker, J. (1981). Instructional practices for promoting skill acquisition and generalization in severely learning disabled adolescents. *Learning Disability Quarterly, 4*, 415-421.
Educational Testing Service. (1989). *A World of Differences: An International Assessment of Mathematics and Science*. Princeton, NJ: Educational Testing Service.
Fennell, F. (1983). Focusing on problem solving in the primary grades. In G. Shufelt & J. Smart (Eds.), *The Agenda in Action*. 1983 NCTM Yearbook (pp. 33-40). Reston, VA: National Council of Teachers of Mathematics.
Fennema, E. (1989, June). *Cognitively Guided Instruction*. Presentation at the ICSEMM Instructional Methods Forum, Washington, D.C.
Fennema, E., Carpenter, T., & Peterson, P. (in press). Learning mathematics with understanding: Cognitively Guided Instruction. In J. Brophy (Ed.), *Advances in Research on Teaching* (Vol.1). Greenwich,CT: JAI.
Fitzmaurice, A. (1980). LD teachers' self-ratings on mathematics education competencies. *Learning Disabilities Quarterly, 3*, 90-94.
Fitzmaurice-Hayes, A. (1984). Curriculum and instructional activities pre-K through grade 2. In J. Cawley (Ed.), *Developmental Teaching of Mathematics for the Learning Disabled* (pp. 95-114). Rockville, MD: Aspen.
Fitzmaurice-Hayes, A. (1985a). Assessment of the severely impaired mathematics student. In J. Cawley (Ed.), *Practical Mathematics Appraisal of the Learning Disabled* (pp. 249-277). Rockville, MD: Aspen.
Fitzmaurice-Hayes, A. (1985b). Classroom implications. In J. Cawley (Ed.), *Cognitive Strategies and Mathematics for the Learning Disabled* (pp. 209-236). Rockville, MD: Aspen.
Fitzmaurice-Hayes, A. (1985c). Whole numbers: Concepts and skills. In J. Cawley (Ed.), *Secondary School Mathematics for the Learning Disabled*, (pp. 83-114). Rockville, MD: Aspen.
Fitzmaurice-Hayes, A. (1989, June). *Teaching Mathematics to Special Education Students*. Presentation at the 1989 ICSEMM Instructional Methods Forum, Washington, D.C.
Fleischner, J., Garnett, K., & Preddy, D. (1982). *Mas. ''y of the Basic Number Facts by Learning Disabled Students: An Intervention Study*. Research Institute for the Study of Learning Disabilities, Teachers College, Columbia University, New York.
Fleischner, J., Nuzum, M., & Marzola, E. (1987). Devising an instructional program to teach arithmetic problem-solving skills to students with learning disabilities. *Journal of Learning Disabilities, 20*, 214-217.
Fleischner, J. & O'Loughlin, M. (1985). Solving story problems: Implications of research for teaching the learning disabled. In J. Cawley, (Ed.), *Cognitive Strategies and Mathematics for the Learning Disabled* (pp. 163-181). Rockville, MD: Aspen.

Fridriksson, T. & Stewart, D. (1988). From the concrete to the abstract: Mathematics for deaf children. *American Annals of the Deaf, 133*, 51-55.
Fuson, K. & Secada, W. (1986). Teaching children to add by counting on with one handed finger patterns. *Cognition and Instruction, 3*, pp. 229-260.
Garnett, K. & Fleischner, J. (1983). Automatization and basic fact performance of normal and learning disabled children. *Learning Disabled Quarterly, 6*, 223-230.
Garofalo, J. (1987). Research report. Metacognition and school mathematics. *Arithmetic Teacher, 34*(May), 22-23.
Garofalo, J. & Lester, F. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education, 16*, 163-176.
Garofalo, J. & Standifer, D. (1989, June). *Report from Working Group One.* Presentation at the ICSEMM Instructional Methods Forum, Washington, D.C.
Ginsburg, H.(1989). *Children's Arithmetic* (2nd ed.). Austin, Texas: PRO-ED.
Goldman, S. (1989). Strategy instruction in mathematics. *Learning Disabilities Quarterly, 12*, 43-55.
Goldman, S., Pellegrino, J., & Mertz, D. (1988). Extended practice of basic addition facts: Strategy changes in learning disabled students. *Cognition and Instruction, 5*, 223-265.
Good, T., Grouws, D., & Ebmeier, H. (1983). *Active Mathematics Teaching.* New York: Longman.
Good, T., Reys, B., Grouws, D., & Mulryan, C. (1989/1990). Using work groups in mathematics instruction. *Educational Leadership, 47*(4), 56-62.
Goodstein, H., Cawley, J., Gordon, S., & Helfgott. (1971). Verbal problem solving among educable mentally retarded children. *American Journal of Mental Deficiency, 76*, 238-241.
Goodstein, H., Bessant, H., Thibodeau, G., Vitello, S., Vlahakos, I. (1972). The effect of three variables on the verbal problem solving of educable mentally handicapped children. *American Journal of Mental Deficiency, 76*, 703-709.
Hasselbring, T., Goin, L., & Bransford, J. (1987). Developing automaticity. *Teaching Exceptional Children, 19*(3), 30-33.
Hasselbring, T., Goin, L., & Bransford, J. (1988). Developing math automaticity in learning handicapped children: The role of computerized drill and practice. *Focus on Exceptional Children, 20*(6), 1-7.
Henderson, R. (1986). Self-regulated learning. Implications for the design of instructional media. *Contemporary Educational Psychology, 11*, 405-427.
Hendrickson, A.D. (1983). Prevention or cure? Another look at mathematics learning problems. In D. Carnine, D. Elkind, A.D. Hendrickson, D. Meichenbaum, R.L. Sieben, & F. Smith (Eds.), *Interdisciplinary Voices in Learning Disabilities and Remedial Education* (pp. 93-107). Austin, TX: Pro-ED.
Herrmann, B. (1989). Characteristics of explicit and less explicit explanations of mathematical problem-solving strategies. *Reading Research and Instruction, 28*(Spring), pp. 1-17.
Hiebert, J. (1984). Children's mathematics learning: The struggle to link form and understanding. *Elementary School Journal, 84*, 497-513.
Hiebert, J. & Wearne, D. (1988). Instruction and cognitive change in mathematics. *Educational Psychologist, 23*, 105-117.
Holmes, E. (1985). *Children Learning Mathematics--A Cognitive Approach to Teaching.* Englewood Cliffs, NJ: Prentice Hall.
Jenkins, M. & Rivera, D. (1989, June). *Report form Working Group Two.* Presentation at the ICSEMM Instructional Methods Forum, Washington, D.C.
Johnston, M., Whitman, T., & Johnson, M. (1981). Teaching addition and subtraction to mentally retarded children: A self-instructional program. *Applied Research in Mental Retardation, 1*, 141-160.
Kennedy, L. (1986). A rationale. *Arithmetic Teacher, 33*(February), 6-7.

Kilpatrick, J. (1985). A retrospective account of the past 25 years of research on teaching mathematical problem solving. In E. Silver (Ed.), *Teaching and Learning Mathematical Problem Solving: Multiple Research Perspectives* (pp. 1-15). Hillsdale, NJ: Erlbaum.

Kirby, J. & Becker, L. (1988). Cognitive components of learning problems in arithmetic. *Remedial and Special Education*, 9(5), 7-16.

Klausmeier, H. & Ripple, R. (1971). *Learning and Human Abilities* New York: Harper and Row.

Kouba, V., Brown, C., Carpenter, T., Lindquist, M., Silver, E., & Swafford, J. (1988a). Results of the fourth NAEP assessment of mathematics: Number, operations, and word problems. *Arithmetic Teacher*, *35*(April), 14-19.

Kouba, V., Brown, C., Carpenter, T., Lindquist, M., Silver, E., & Swafford, J. (1988b). Results of the fourth NAEP assessment of mathematics: Measurement, geometry, data interpretation, attitudes and other topics. *Arithmetic Teacher*, *33*(May), 10-16.

Leon, J. & Pepe, H. (1983). Self instructional training. Cognitive behavior modification for remediating arithmetic deficits. *Exceptional Children*, *50*, 54-60.

Lester, F. (1985). Methodological considerations in research on mathematical problem solving instruction. In E. Silver (Ed.), *Teaching and Learning Mathematical Problem Solving: Multiple Research Perspectives* (pp. 41-69). Hillsdale, NJ: Erlbaum.

Lester, F. & Garofalo, J. (1987, April). *The Influence of Affects, Beliefs, and Metacognition on Problem Solving Behavior: Some Tentative Speculations*. Paper presented at the annual meeting of the American Educational Research Association, Washington, D.C. (ED 281 758).

Lindquist, M. (1987). Strategic teaching in mathematics. In B. Jones, A. Palincsar, D. Ogle, & E. Carr (Eds.), *Strategic Teaching and Learning: Cognitive Instruction in the Content Areas* (pp. 111-134). Alexandria, VA: ASCD.

Lloyd, J. & Keller, C. (1989). Effective mathematics instruction: Development, instruction, and programs. *Focus on Exceptional Children*, *21*(7), 1-10.

Lloyd, J., Saltzman, N., & Kauffman, J. (1981). Predictable generalizations in academic learning as a result of preskills and strategy training. *Learning Disability Quarterly*, *4*, 203-216.

Marten, B. (1989, June). *CGI in the Classroom*. Presentation at the ICSEMM Instructional Methods Forum. Washington, D.C.

Martin, P. & Carnahan, S. (1989). Teaching hands-on science and math. What hinders and what helps. *Connect* (March), 1-4.

Maurer, S. (1987). New knowledge about errors and new views about learners. What they mean to educators and more educators would like to know. In A. Schoenfeld (Ed.), *Cognitive Science and Mathematics Education* (pp. 165-187). Hillsdale, NJ: Erlbaum.

Mayer, R. (1982). The psychology of mathematical problem solving. In F. Lester & J. Garofalo (Eds.), *Mathematical Problem Solving: Issues in Research* (pp. 1-13). Philadelphia: Franklin Institute.

Mayer, R. (1985). Implications of cognitive psychology for instruction in mathematical problem solving. In E. Silver (Ed.), *Teaching and Learning Mathematical Problem Solving: Multiple Research Perspectives* (pp. 123-138). Hillsdale, NJ: Erlbaum.

McLeod, T. & Armstrong, S. (1982). Learning disabilities in mathematics--skill deficits and remedial approaches. *Learning Disability Quarterly*, *5*, 305-311.

Meyen, E. (1968). An investigation of age placement difficulty and importance of basic skills curriculum of EMR students. Unpublished doctoral dissertation, University of Iowa.

Montague, M. (in press). Strategy instruction & mathematical problem solving. *Journal of Reading, Writing, and Learning Disabilities*.

Montague, M. & Bos, C. (1986). The effects of cognitive strategy training on verbal math problem solving performance of learning disabled adolescents. *Journal of Learning Disabilities, 19*, 26-33.

Montague, M. & Bos, Candace. (in progress). Cognitive and metacognitive characteristics of eighth grade students' mathematical problem solving.

National Council of Teachers of Mathematics. (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Research Council (1989). *Everybody Counts*. Washington, D.C.: National Academy Press.

Palincsar, A. (1986). Metacognitive strategy instruction. *Exceptional Children, 53*, 118-124.

Payne, J., Polloway, E., Smith, J., & Payne, R. (1981). *Strategies for Teaching the Mentally Retarded*, 2nd ed. Columbus, Ohio: Merrill.

Pellegrino, J. & Goldman, S. (1987). Information processing and elementary mathematics. *Journal of Learning Disabilities, 20*, 23-32.

Peters, E., Lloyd, J., Hasselbring, T., Goin, L., Bransford, J., & Stein, M. (1987). Effective mathematics instruction. *Teaching Exceptional Children, 19*(3), 30.

Peterson, P., Fennema, E., & Carpenter, T. (1988/1989). Using knowledge of how students think about mathematics. *Educational Leadership, 46*(4), 42-46.

Peterson, P., Fennema, E., Carpenter, T., & Loef, M. (1989). Teaching pedagogical content beliefs in mathematics. *Cognition and Instruction, 6*, 1-40.

Peterson, S., Mercer, C., & O'Shea, L. (1988). Teaching learning disabled students place value using concrete to abstract sequence. *Learning Disabilities Research, 4*, 52-56.

Pressley, M. (1986). The relevance of good strategy user model to the teaching of mathematics. *Educational Psychologist, 21*(1-2), 139-161.

Raphael, D. & Wahlstrom, M. (1989). The influence of instructional aids on mathematics achievement. *Journal for Research in Mathematics Education, 20*, 173-190.

Resnick, L. (1987). *Education and Learning to Think*. Committee on Mathematics, Science, and Technology Education. Washington, D.C.: National Academy Press. (ED 289 832).

Resnick, L. & Omanson, S. (1986). Learning to understand arithmetic. In R. Glaser (Ed.), *Advances in Instructional Psychology, Vol. 3* (pp. 41-95). Hillsdale, NJ: Erlbaum.

Riley, M. (1981). Conceptual and procedural knowledge in development. Unpublished doctoral thesis, University of Pittsburgh.

Riley, M., Greeno, J., & Heller, J. (1983). Development of children's problem solving ability. In H. Ginsburg (Ed.), *Development of Mathematical Thinking* (pp. 153-196). New York: Academic Press.

Rivera, D. & Smith, D. (1987). Influence of modeling on acquisition and generalization of computational skills: A summary of research from three sites. *Learning Disabilities Quarterly, 10*, 69-80.

Romberg, T. & Carpenter, T. (1986). Research on teaching and learning mathematics. Two disciplines of scientific inquiry. In M. Wittrock (Ed.), *Handbook of Research on Teaching, 3rd ed.* (pp. 850-873). New York: MacMillan.

Russell, R. & Ginsburg, H. (1984). Cognitive analysis of children's mathematical difficulties. *Cognition and Instruction, 1*, 217-244.

Scheid, K. (1989). *Cognitive and Metacognitive Learning Strategies--Their Role in the Instruction of Special Education Students*. Columbus, Ohio:Linc Resources.

Schenk, W. (1973). Pictures and the indefinite quantifier in verbal problem solving among EMR children. *American Journal of Mental Deficiency, 78*, 272-276.

Schoenfeld, A. (1982). Some thoughts on problem solving research and mathematics education. In F. Lester & J. Garofalo (Eds.), *Mathematical Problem Solving--Issues in Research* (pp. 27-37). Philadelphia: Franklin Institute.

Schoenfeld, A. (1987). What's all this fuss about metacognition. In A. Schoenfeld (Ed.), *Cognitive Science and Mathematics Education* (pp. 189-215). Hillsdale, NJ: Erlbaum.
Schoenfeld, A. (1988). When good teaching leads to bad results. *Educational Psychologist*, *23*, 145-166.
Schumaker, J. & Hazel, S. (1984). Social skills assessment and training for the learning disabled: Who's on first and what's on second? Part I. *Journal of Learning Disabilities*, *17*, 422-431.
Schunk, D. (1981). Modeling and attributional effects on children's achievements--a self-efficacy analysis. *Journal of Educational Psychology*, *73*, 93-105.
Schunk, D. & Cox, P. (1986). Strategy training and attributional feedback with learning disabled students. *Journal of Educational Psychology*, *78*, 201-209.
Secada, W., Fuson, K., & Hall, J. (1983). The transition from counting all to counting on in addition. *Journal for Research in Mathematics Education*, *14*, 47-57.
Share, D., Moffitt, T., & Silva, P. (1988). Factors associated with arithmetic and reading disability. *Journal of Learning Disabilities*, *21*, 313-320.
Shufelt, G. (Ed.) (1977). Etcetera. *Arithmetic Teacher*, *25*(December), p.37.
Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4-15.
Silver, E. (1985). Research in teaching mathematical problem solving: Some underrepresented themes and needed directions. In E.Silver (Ed.), *Teaching and Learning Mathematical Problem Solving: Multiple Research Perspectives* (pp. 247-266). Hillsdale, NJ: Erlbaum.
Silver, E. (1987). Foundations of cognitive theory and research for mathematics problem solving instruction. In A. Schoenfeld (Ed.), *Cognitive Science and Mathematics Education* (pp. 33-60). Hillsdale, NJ: Erlbaum.
Slavin, R. & Karweit, N. (1985). Effects of whole class, ability groups, and individualized instruction on mathematics achievement. *American Educational Research Journal*, *22*, 351-367.
Slavin, R., Madden, N., & Leavey, M. (1984). Effects of team assisted individualization on the mathematics achievement of academically handicapped and non-handicapped students. *Journal of Educational Psychology*, *76*, 813-819.
Slife, B., Weiss, J., & Bell, T. (1985). Separability of metacognition and cognition. Problem solving in learning disabled and regular students. *Journal of Educational Psychology*, *77*, 437-445.
Swing, S., Stoiber, K., & Peterson, P. (1988) Thinking skills vs. learning time: Effects of alternative classroom-based interventions on students' mathematics problem solving. *Cognition and Instruction*, *5*, 123-191.
Thackwray, D., Meyers, A., Schleser, R., & Cohen, R. (1985). Achieving generalization with general versus specific self-instruction: Effects on academically deficient children. *Cognitive Therapy and Research*, *9*, 297-308.
Thompson, C. & Rathmell, E. (1988). NCTM's standards for school mathematics, K-12. *Arithmetic Teacher*, *35*(May), 17-19.
Thornton, C. (1978). Emphasizing thinking strategies in basic fact instruction. *Journal for Research in Mathematics Education*, *9*, 213-227.
Thornton, C. (1989a, June). *Cognitive Based Methods for Enhancing the Mathematical Problem-Solving Capabilities of Special Education Students*. Presentation at the ICSEMM Instructional Methods Forum, Washington, D.C.
Thornton, C. (1989b). "Look ahead" activities spark success in addition and subtraction number fact learning. *Arithmetic Teacher*, *36*(April), 8-11.
Thornton. C., Jones, G., & Toohey, M. (1983). A multisensory approach to thinking strategies for remedial instruction in basic addition facts. *Journal for Research in Mathematics Education*, *14*, 198-203.
Thornton, C. & Smith, P. (1988). Action research: Strategies for learning subtraction facts. *Arithmetic Teacher*, *35*(April), 8-12.

Thornton, C. & Toohey, M. (1985). Basic math facts: Guidelines for teaching and learning. *Learning Disabilities Focus, 1*, 44-57.

Thornton, C. & Toohey, M. (1986). Subtraction facts hide-and-seek cards can help. *Teaching Exceptional Children, 19*, 10-14.

Thornton, C., Tucker, B., Dossey, J., & Bazik, E. (1983) *Teaching Mathematics to Children with Special Needs*. Menlo Park, CA: Addison-Wesley.

Thornton, C. & Wilmot, B. (1986). Special learners. *Arithmetic Teacher, 33*(February), 38-41.

Trafton, P. (1984). Toward more effective, efficient instructions in mathematics. *Elementary School Journal, 84*, 514-528.

Van Lehn, K. (1983). On the representation of procedures in repair theory. In H. Ginsburg (Ed.), *The Development of Mathematical Thinking* (pp. 197-252). New York: Academic Press.

Whitman, T. & Johnston, M. (1983). Teaching addition and subtraction with regrouping to educable mentally retarded children. A group self-instructional training program. *Behavior Therapy, 14*, 127-143.

Zhu, X. & Simon, H. (1987). Learning mathematics from examples by doing. *Cognition and Instruction, 4*, 137-166.

Zigler, E., Balla, D., & Hodapp, R. (1984). On definition and classification of mental retardation. *American Journal on Mental Deficiency, 89*, 215-230.

Information Center

for SPECIAL EDUCATION MEDIA and MATERIALS

LINC Resources Inc.

4820 Indianola Avenue

Columbus, Ohio 43214

(800) 772-7372

(614) 885-5599

Appendix 16

END

U.S. Dept. of Education

Office of Education
Research and
Improvement (OERI)

ERIC

Date Filmed

March 21,1991